CS 70 Discrete Mathematics and Probability Theory
Spring 2017 Rao

DIS 2b

1 Trees

Recall that a *tree* is a connected acyclic graph (graph without cycles). In the note, we presented a few other definitions of a tree, and in this problem, we will prove two fundamental properties of a tree, and derive two definitions of a tree we learn from lecture note based on these properties. Let's start with the properties:

(a) Prove that any pair of vertices in a tree are connected by exactly one (simple) path.

(b) Prove that adding any edge between two vertices of a tree creates a simple cycle.

Now you will show that if a graph satisfies either of these two properties then it must be a tree:

(c) Prove that if every pair of vertices in a graph are connected by exactly one simple path, then the graph must be a tree.

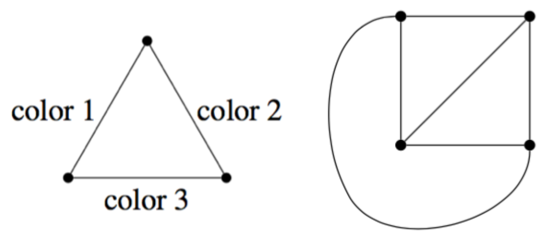
(d) Prove that if the graph has no simple cycles and has the property that the addition of any single edge (not already in the graph) will create a simple cycle, then the graph is a tree.

2 Hamiltonian Tour in a Hypercube

An alternative type of tour to an Eulerian Tour in graph is a Rudrata Tour: a tour that visits every vertex exactly once. Prove or disprove that the hypercube contains a Rudrata cycle, for hypercubes of dimension $n \geq 2$.

3 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.



(a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers $1,2,3$ for colors. A figure is shown on the right.)

(b) How many colors are required to edge color a 3-dimensional hypercube?

(c) Prove that any graph with maximum degree d can be edge colored with $2d-1$ colors.

(d) Show that any tree has a degree 1 vertex. (You may use any definition of a tree that we provided in the notes, homeworks or lectures to prove this fact.)

(e) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.