RSA Encryption

CS70 - Spring 2017

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UC Berkeley

Agenda

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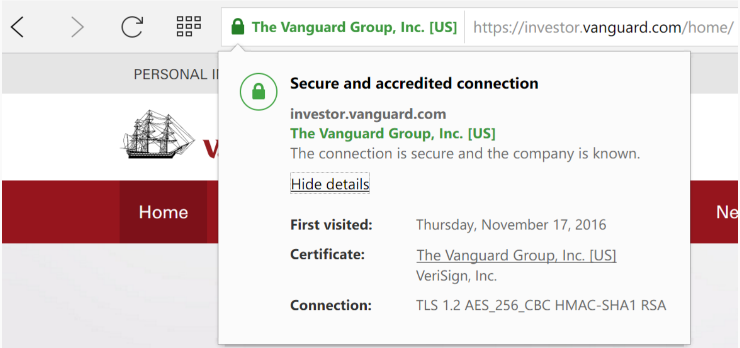

Motivation

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Unfortunately, real life looks more like this. How do I form a private channel to the bank if there's someone snooping on my connection?

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One-Time Pad

Simple encryption scheme (”one time pad”): given a *plaintext* we want to encrypt (e.g. SSN, represented as a bitstring) and a *key* of equal length, xor each bit of the plaintext with the corresponding bit of the key to get a *ciphertext*.

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How do we decrypt? Notice that $x \oplus y \oplus x = y \oplus x \oplus x = y \oplus 0 = y$. So: just xor the ciphertext with the key, bitwise, to get plaintext back.

Example/Live Demo

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Needs a key to be shared before the transmission is done. If I need to walk into bank to share a secret key before sending them my SSN, why not just give my SSN to them when I walk in?

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Big idea: the bank gives everyone a mathematical safe that they can put stuff into, but only the bank can unlock.

The RSA Algorithm

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Theorem: For the encryption/decryption protocol on the previous slide, $D(E(x)) = x \pmod{N}$ for all $x \in \{0, 1, \ldots n-1\}$.

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Argument for q is exactly the same. Therefore $p|(x^{ed} - x)$. ■

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How do we test for primality efficiently? Lots of tests that will tell you "this is definitely not a prime" or "this may or may not be a prime" very quickly - simplest is based on Fermat's little theorem! Efficient algorithm for distinguishing between "this is not a prime" and "this definitely is a prime" was found in 2002 by Agrawal, Kayal, Saxena - major breakthrough!

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$$51^{32} \cdot 51^8 \cdot 51^2 \cdot 51^1 = (60) * (53) * (60) * (51) \equiv 2 \pmod{77} \ .$$

Implementation Concerns: Repeated Squaring II

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How many multiplications required? $O(\log y)$. Much faster than multiplying y times!

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The security RSA, like all almost all encryption schemes, relies on *hardness assumptions*. We need to assume something is hard in order to show that decrypting something, or even getting some information about the plaintext, *even with full information*, is hard.

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Security of breaking RSA requires on hardness of factoring large integers.

RSA in Practice: Padding

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To address this: "pad" the plaintext by appending extra junk bits to it to make it longer. Determining which junk bits would be secure is not trivial![1]

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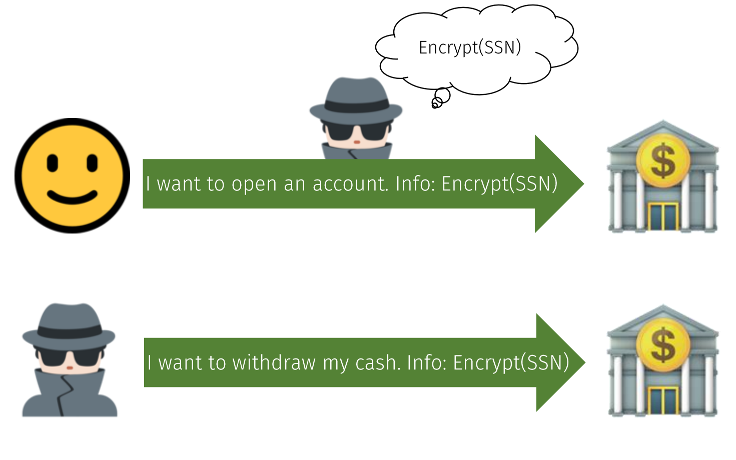
RSA in Practice: Replay Attacks, Nonces

Replay attack: if someone know your ciphertext, he can always send it again...



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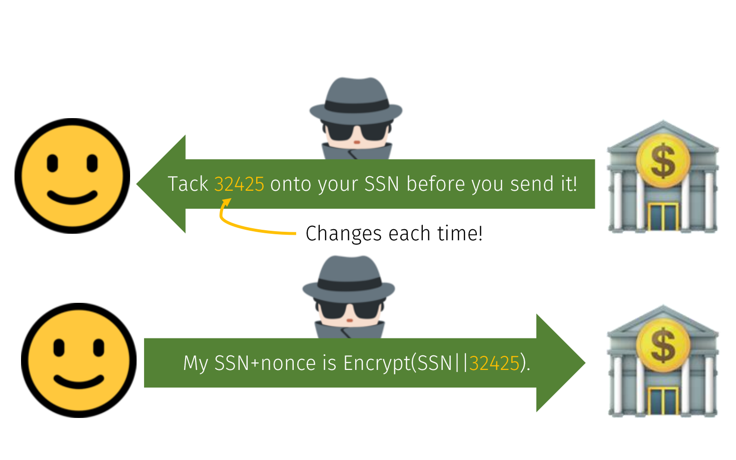

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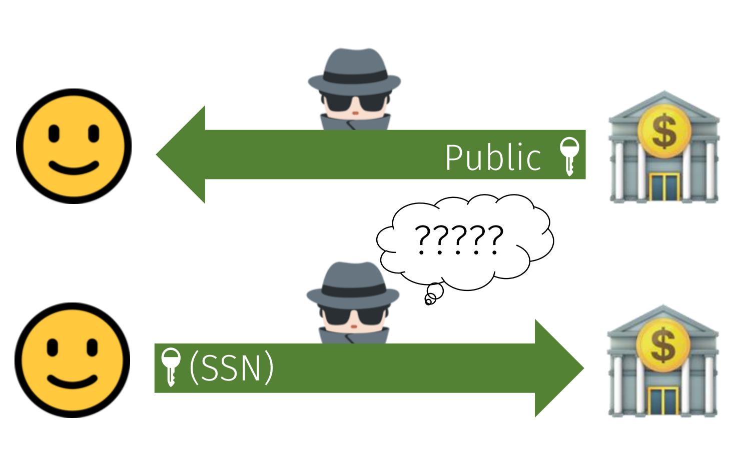
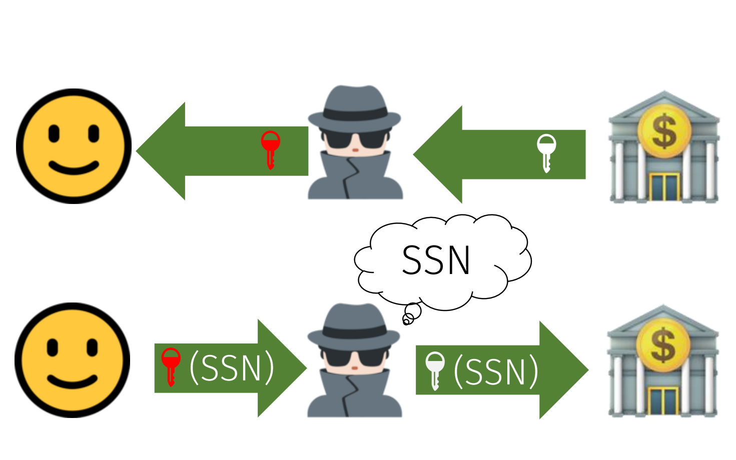


RSA in Practice: MITM

RSA allows you to protect your communication from snooping. It does **not** protect your communication from tampering ("man in the middle", or MITM attacks).



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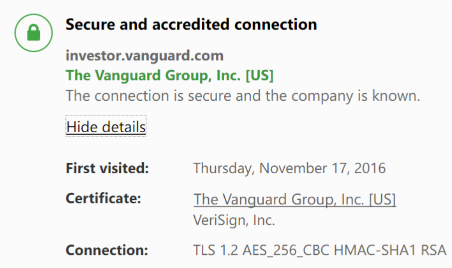

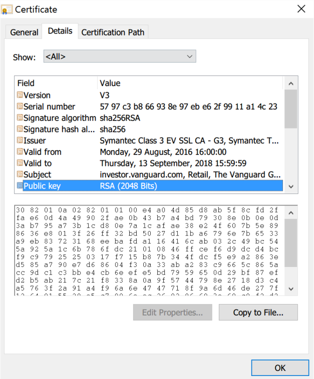

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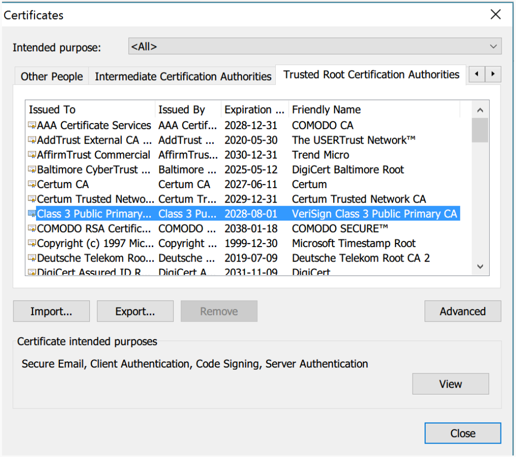
What about security? Making the signature requires computing $D(C, k_v)$ which is hard without k_v. Same security analysis of RSA applies!

Whom do you trust?

You need to trust the browser vendor/computer manufacturer that gave you the list of trusted CAs initially *and* trust the CA to only sign legitimate certificates.



What happens when trust breaks down?

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TNW

Superfish admits installing root certificate authority to show ads on secure sites

by OWEN WILLIAMS — Feb 20, 2015 in INSIDER

ars technica

RISK ASSESSMENT / SECURITY & HACKTIVISM

Dell does a Superfish, ships PCs with easily cloneable root certificates

Root certificate debacle that hit Lenovo now visits the House of Dell.

What happens when trust breaks down?

Bugzilla@Mozilla

New Account | Log In | Forgot Password

mozilla

Home New Browse Search

Search [help] Reports

Product Dashboard

Bug 1311713 - CA Comodo used broken OCR and issued certificates to the wrong people

Last Comment

Debian Bug report logs - #744027

Please remove StartCom Certification Authority root certificate

Questions?