Lecture 5: Graphs.

Graphs!

Lecture 5: Graphs.

Graphs!
Euler

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!

Lecture 5: Graphs.

Graphs!

- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Lecture 5: Graphs.

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Lecture 5: Graphs.

- Graphs!
  - Euler
  - Definitions: model.
  - Fact!
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- Planar graphs.

Lecture 5: Graphs.

- Graphs!
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  - Definitions: model.
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  - Euler Again!!
- Planar graphs.
  - Euler Again!!!!

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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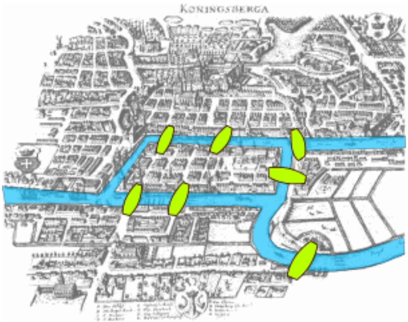

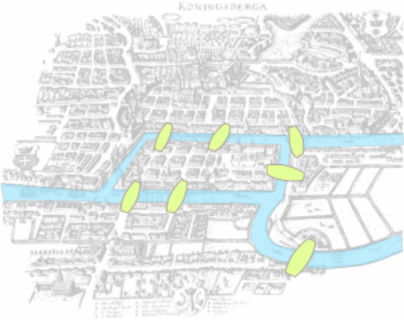

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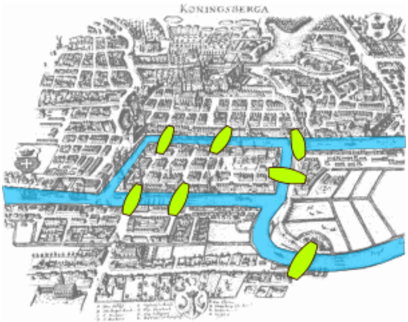

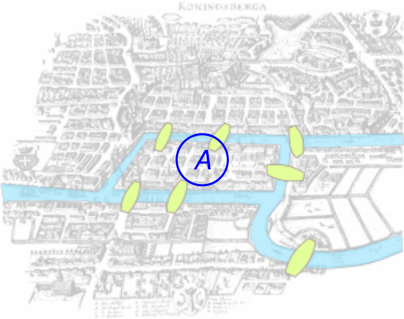

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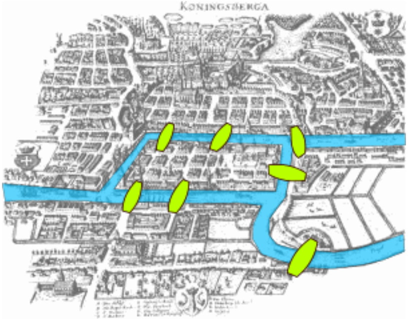

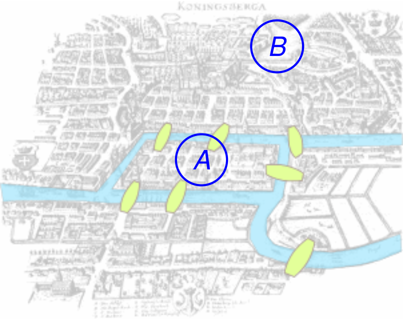

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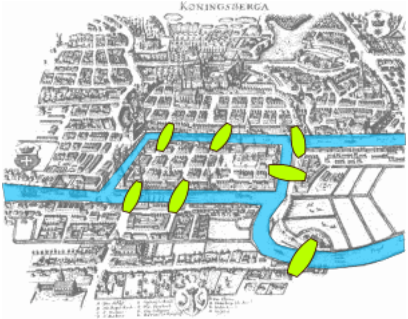

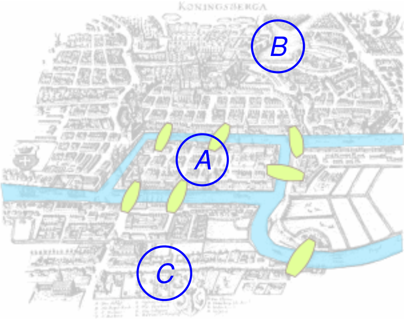

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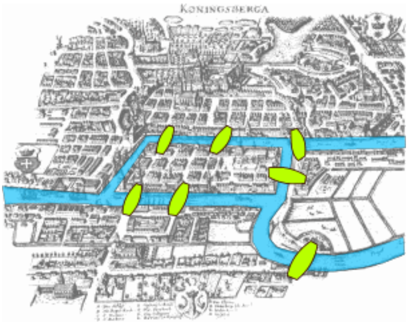

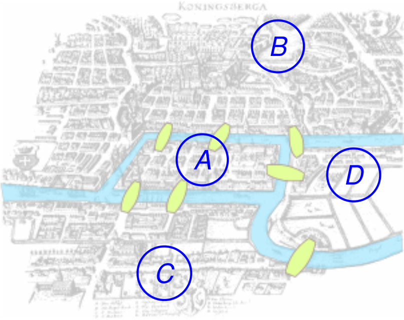

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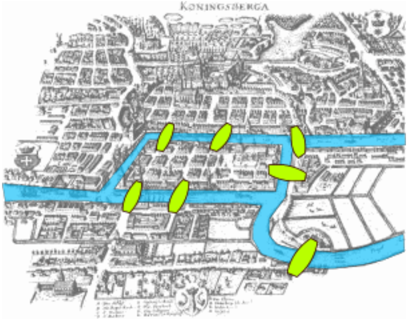

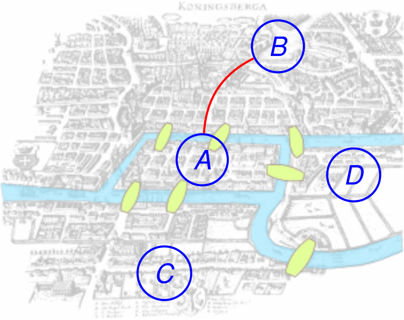

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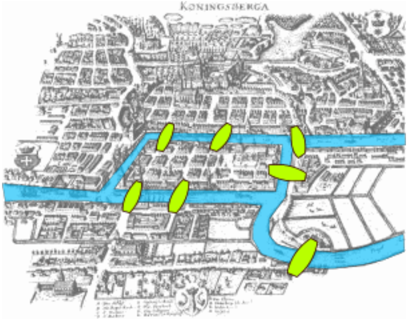

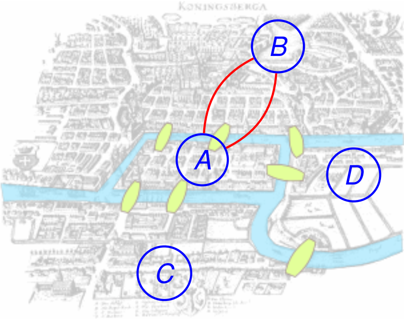

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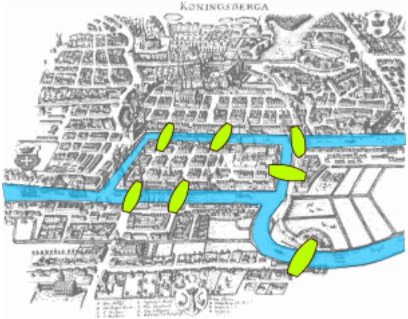

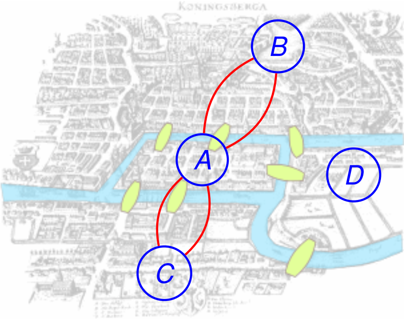

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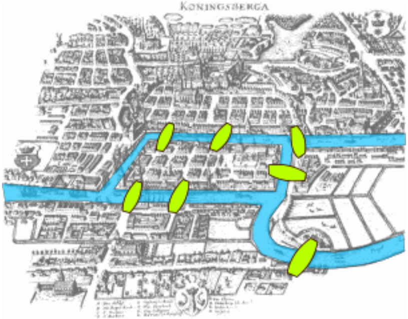

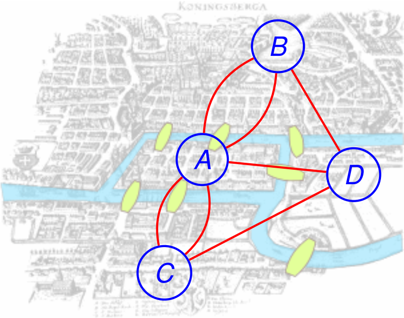

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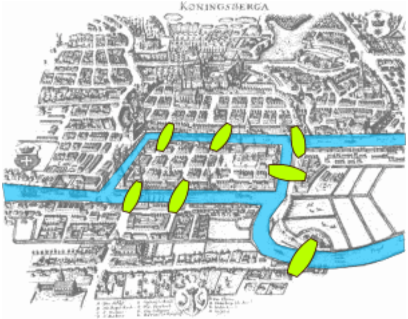

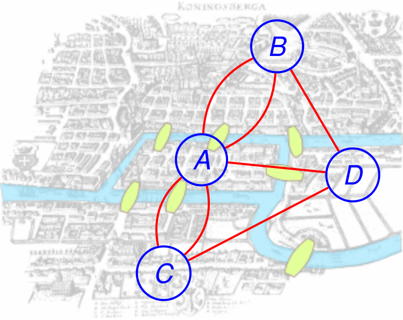

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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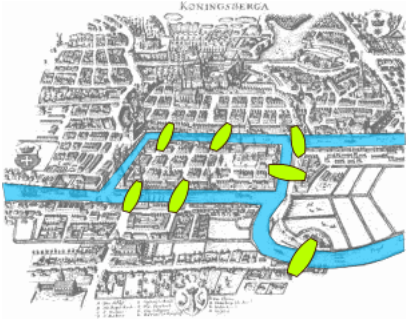
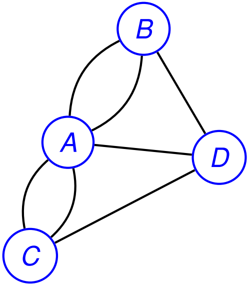
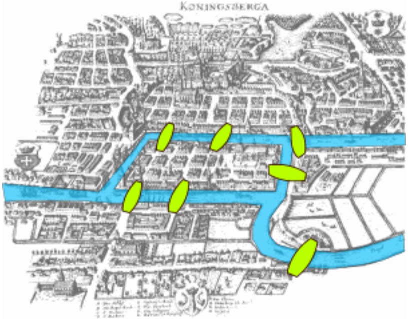


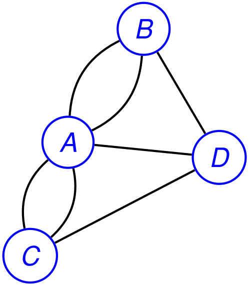


Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?





Can you draw a tour in the graph where you visit each edge once?
Yes? No?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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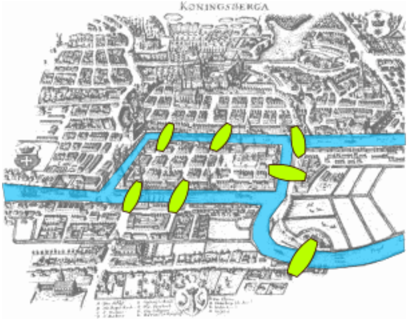
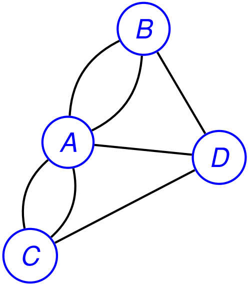
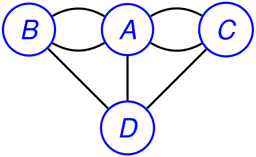




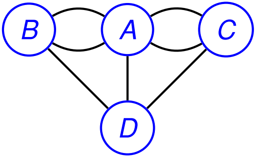
Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Graphs: formally.



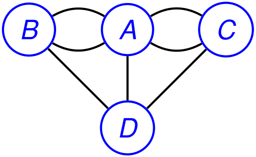
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

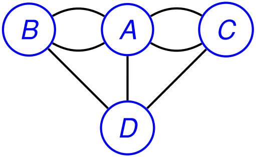
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

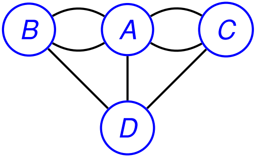


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



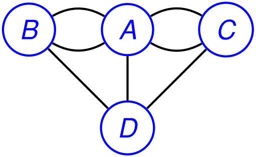
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



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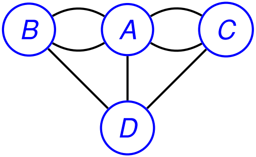


Graph: $G = (V, E)$.

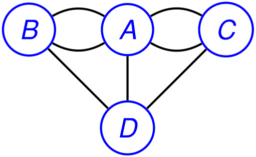
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

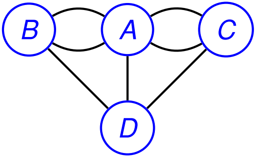
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

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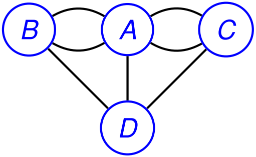

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$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

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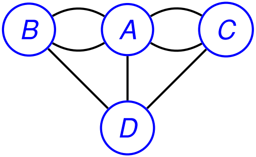

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V - set of vertices.

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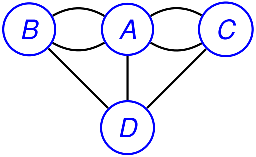
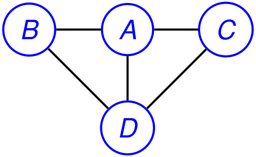
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

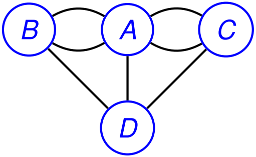
$E \subseteq V \times V$ - set of edges.

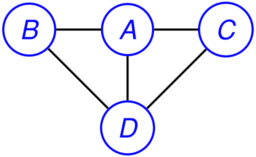
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

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$E \subseteq V \times V$ - set of edges.

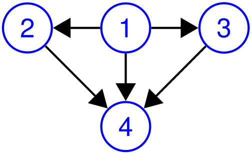
$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

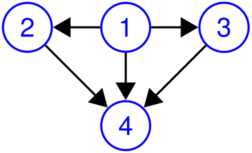
Multigraph above.

Directed Graphs



$G = (V, E)$.

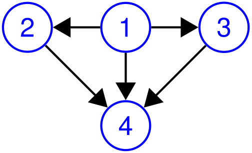
Directed Graphs



$G = (V, E)$.

V - set of vertices.

Directed Graphs

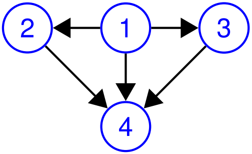


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



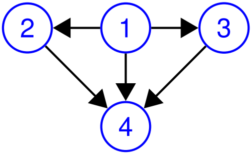
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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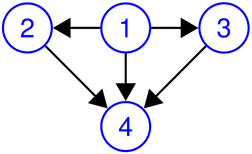
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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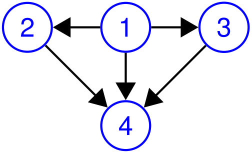
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



$G = (V, E)$.

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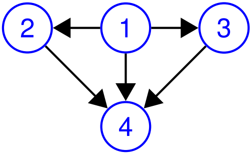
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4),$

Directed Graphs



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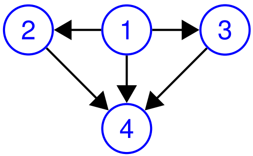
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

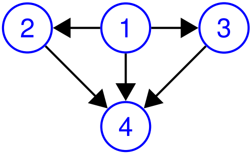
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



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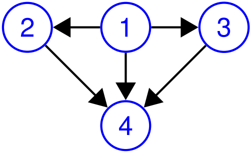
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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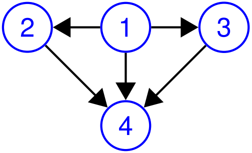
E ordered pairs of vertices.
$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2,

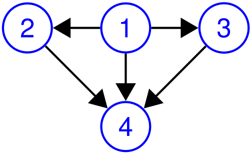
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
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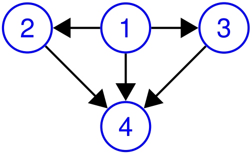
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

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E ordered pairs of vertices.

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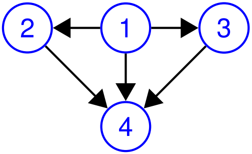

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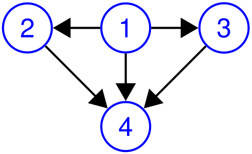
Social Network:

Directed Graphs



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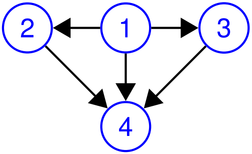

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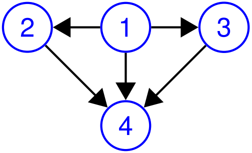

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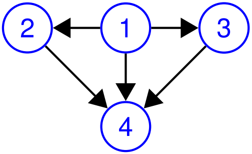
Friends.

Directed Graphs



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Social Network: Directed? Undirected?
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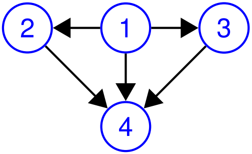
Directed Graphs



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Social Network: Directed? Undirected?
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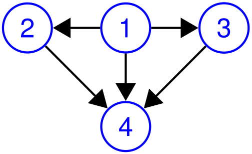
Likes.

Directed Graphs



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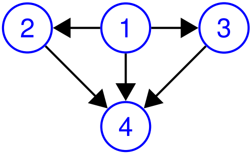
Directed Graphs



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Graph Concepts and Definitions.

Graph: $G = (V, E)$

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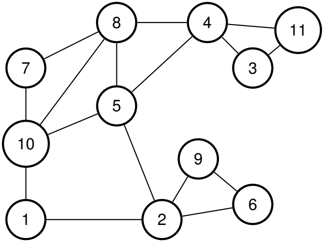

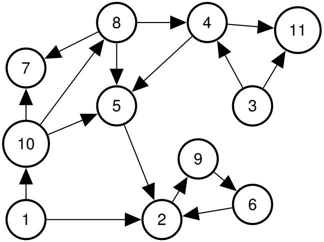

Neighbors of 10?

Graph Concepts and Definitions.

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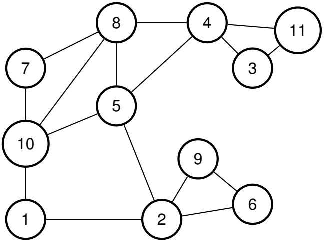
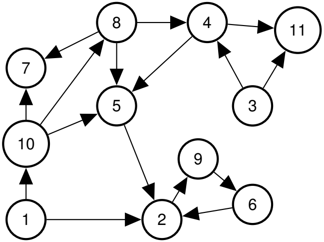
neighbors, adjacent, degree, incident, in-degree, out-degree





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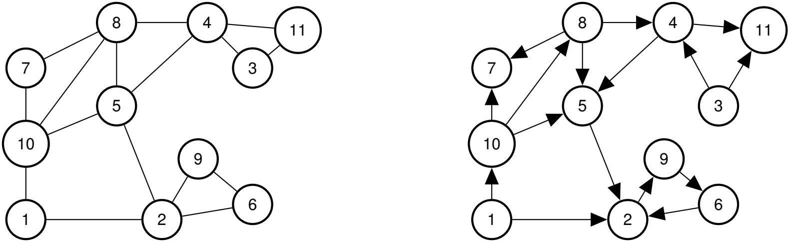

Neighbors of 10? 1,5,

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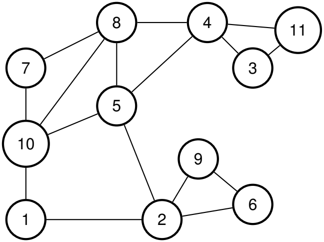

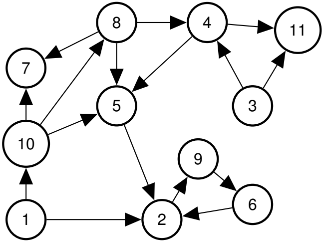

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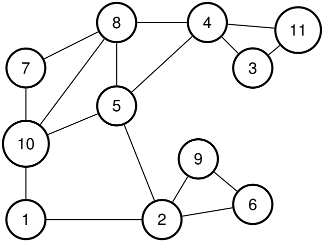

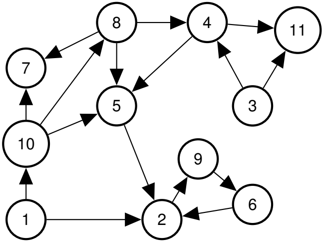

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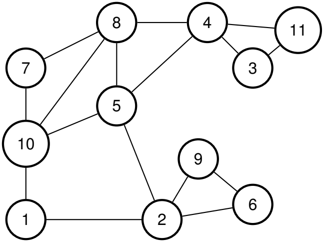

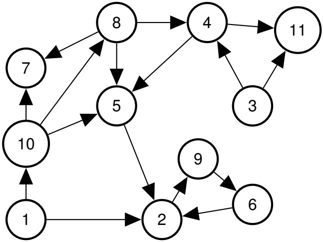

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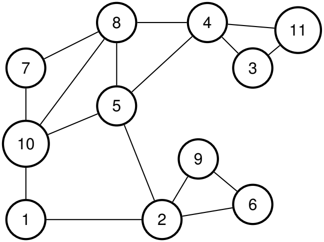

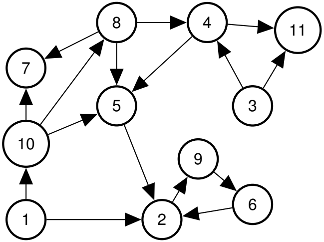

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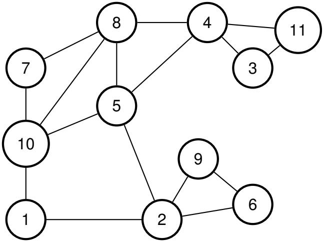

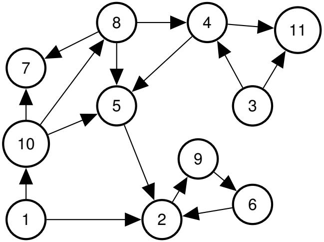

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Degree of vertex 1?

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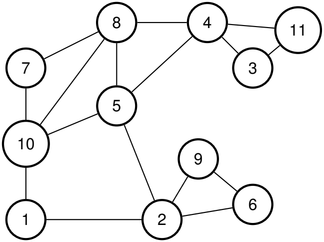

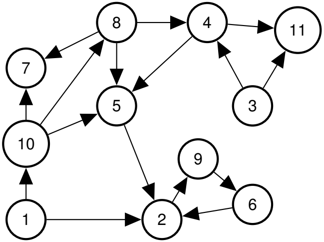

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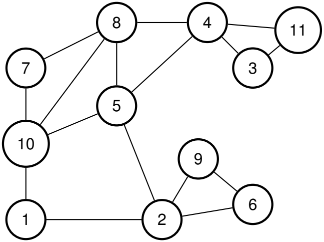

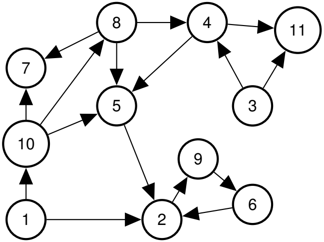

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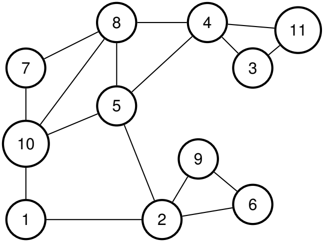

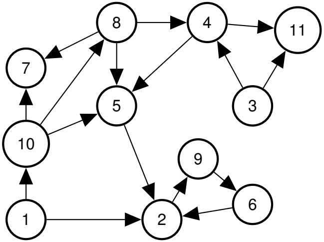

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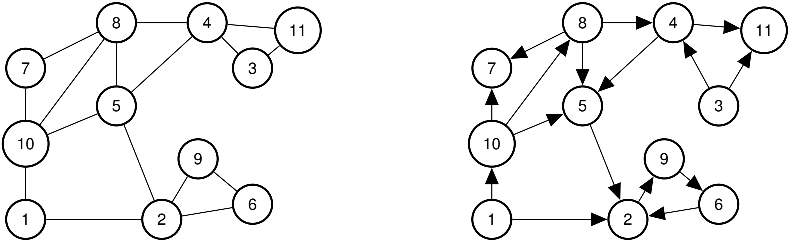

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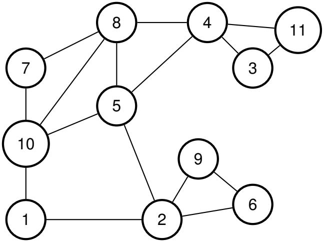

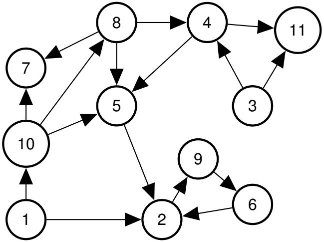

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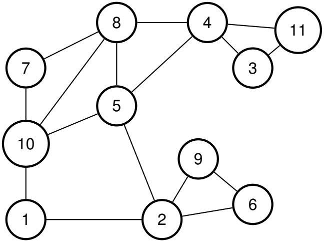

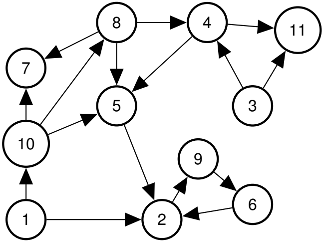

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In-degree of 10?

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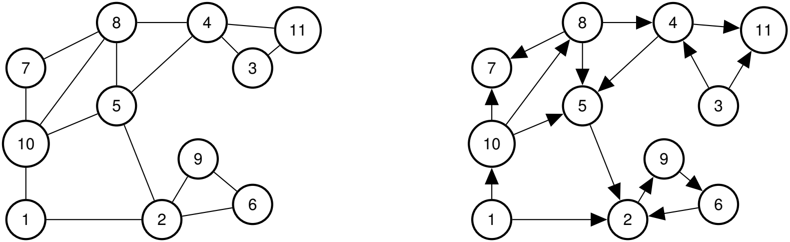
neighbors, adjacent, degree, incident, in-degree, out-degree



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In-degree of 10? 1

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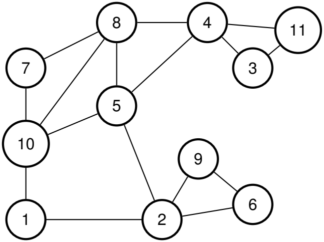

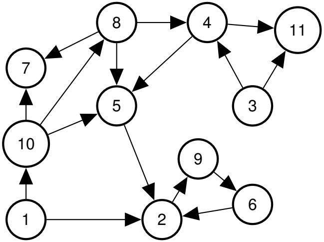

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In-degree of 10? 1 Out-degree of 10?

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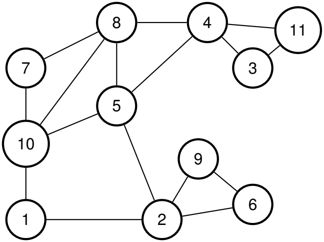

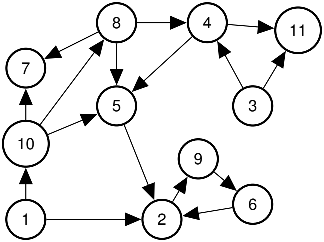

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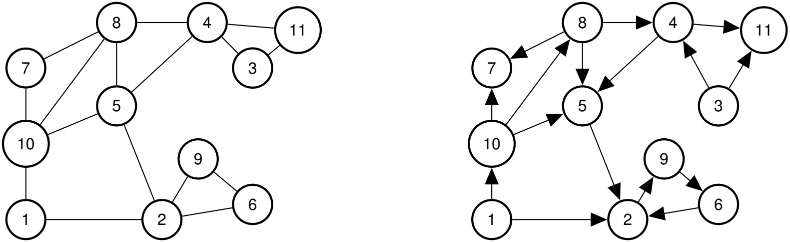

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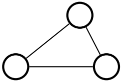

Not (B)!

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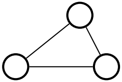

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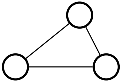

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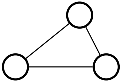
Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



Not (B)! Triangle.

What?

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

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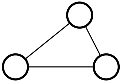

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Quick Proof.

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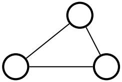
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...

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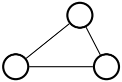

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..

Quick Proof.

The sum of the vertex degrees is equal to

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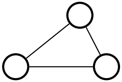

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

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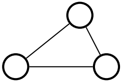

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Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute?

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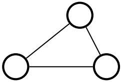

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.

Quick Proof.

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(C) What?

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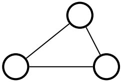

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!

Quick Proof.

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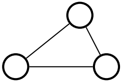

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$2|E|$ incidences are contributed in total!
What is degree v?

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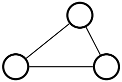

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How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!
What is degree v? incidences contributed to v!

Quick Proof.

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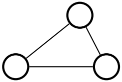

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sum of degrees is total incidences

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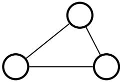

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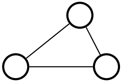

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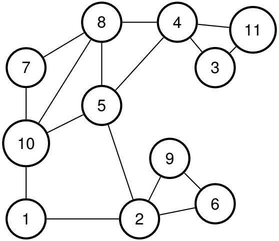
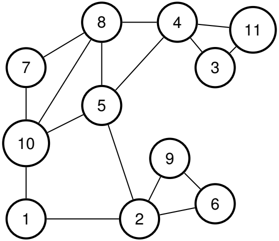
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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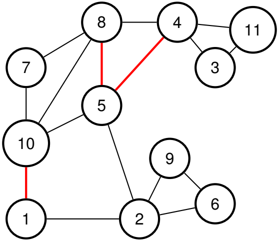
Path?

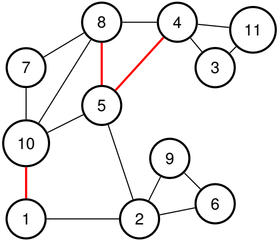
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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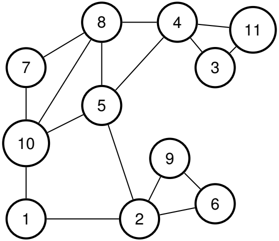

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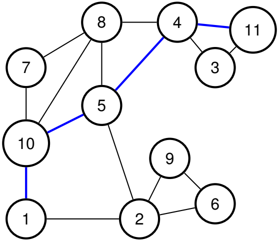
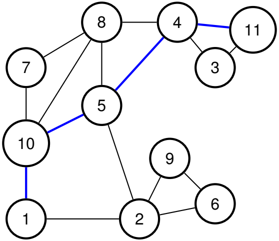
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$?

Paths, walks, cycles, tour.



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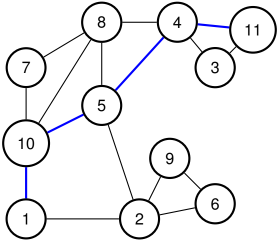

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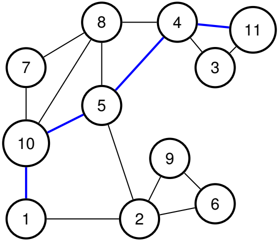
Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

Paths, walks, cycles, tour.



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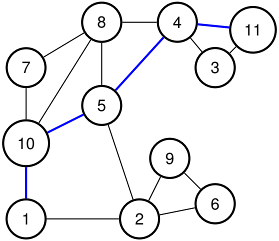
Quick Check!

Paths, walks, cycles, tour.



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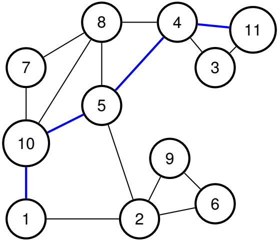
Quick Check! Length of path?

Paths, walks, cycles, tour.



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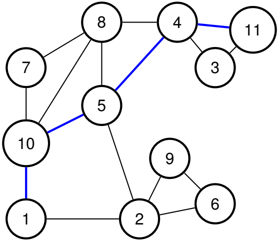
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

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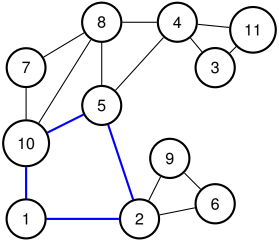
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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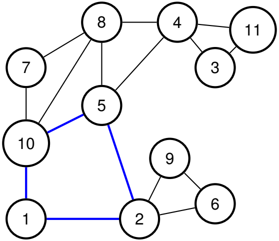
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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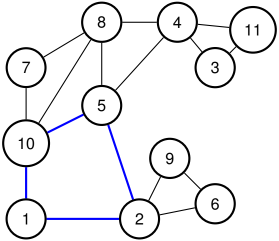
Cycle: Path with $v_1 = v_k$. Length of cycle?

Paths, walks, cycles, tour.



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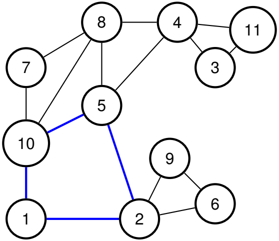
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Paths, walks, cycles, tour.



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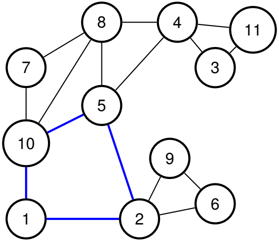
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Path is usually simple.

Paths, walks, cycles, tour.



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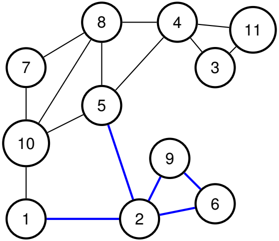
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple. No repeated vertex!

Paths, walks, cycles, tour.



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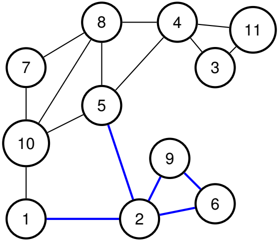
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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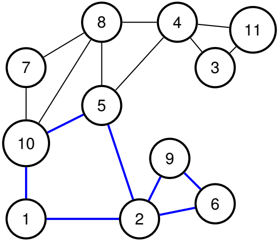

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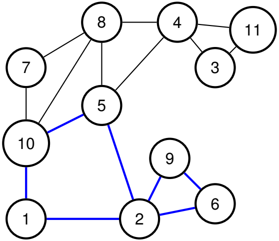
Tour is walk that starts and ends at the same node.

Paths, walks, cycles, tour.



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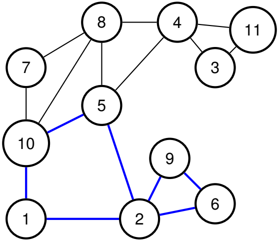
Paths, walks, cycles, tour.



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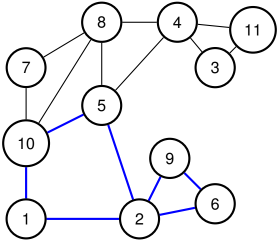
Paths, walks, cycles, tour.



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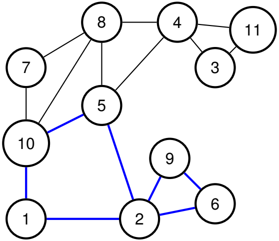
Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple. No repeated vertex!

Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

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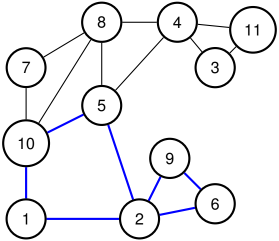

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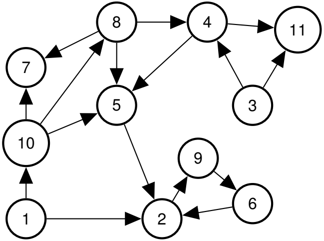

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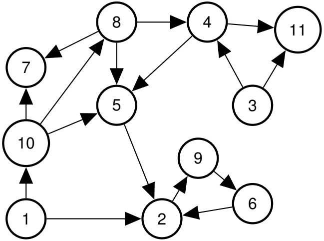

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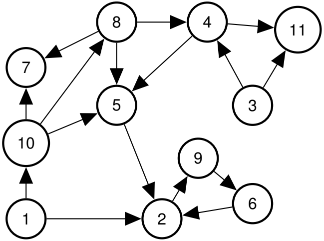

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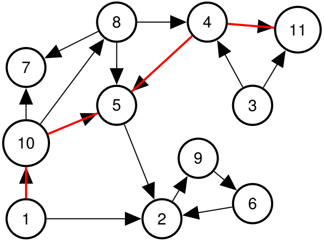

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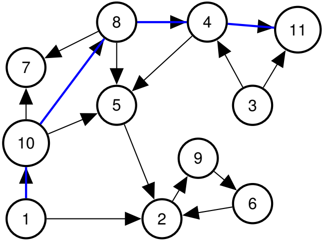

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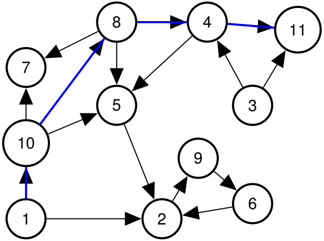

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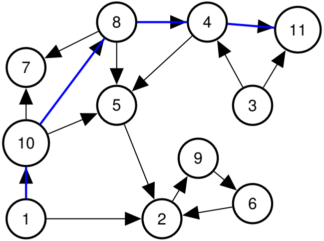

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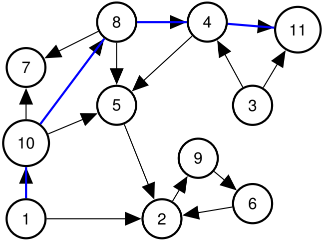

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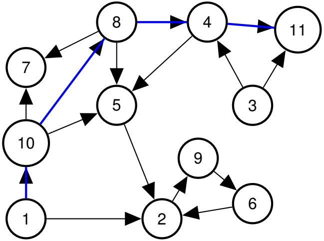

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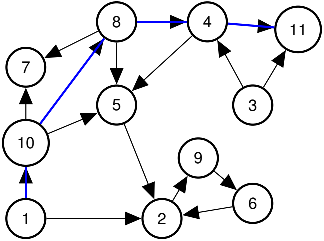

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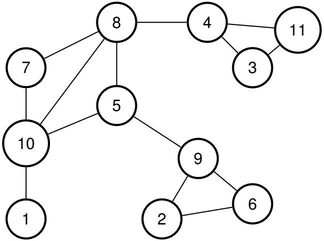
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity



u and *v* are connected if there is a path between *u* and *v*.

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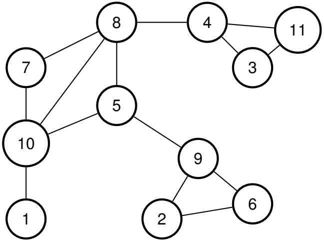

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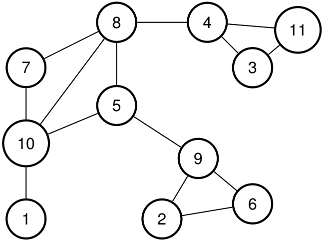

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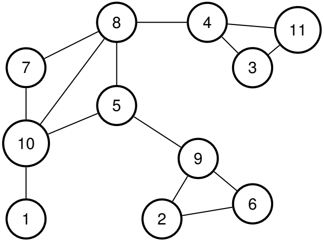

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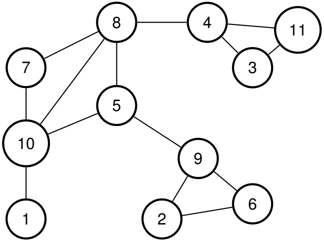

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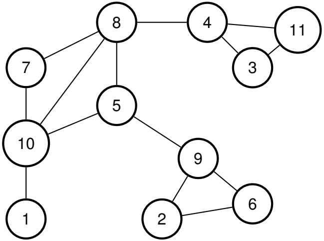

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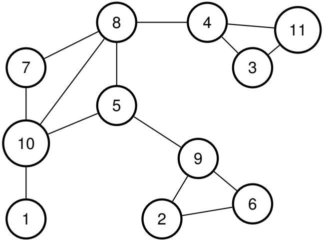

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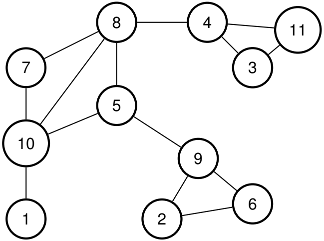

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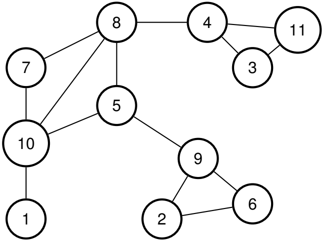

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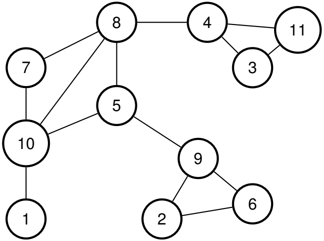

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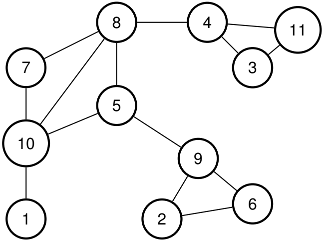

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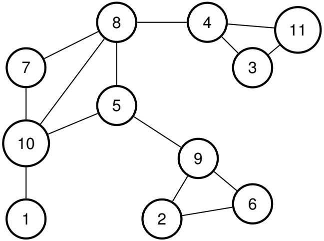

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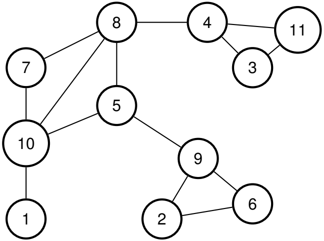

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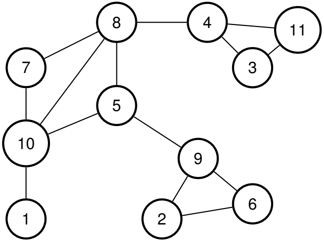

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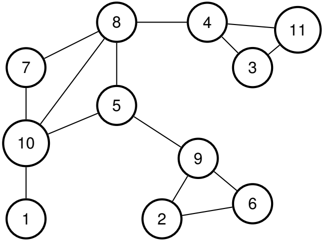
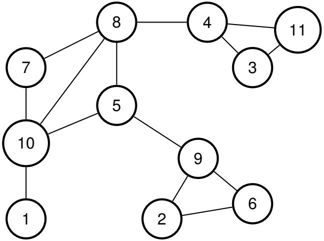


Is graph above connected?



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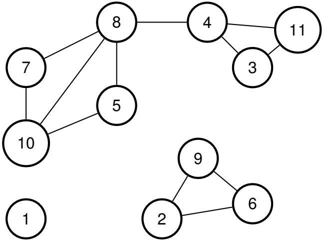

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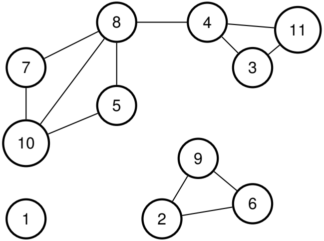
How about now?



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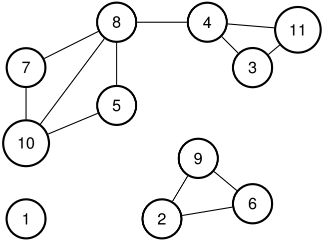

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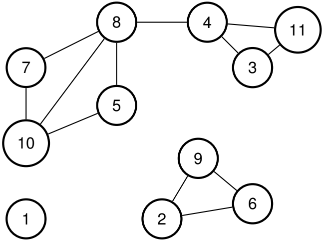
Connected Components?



Is graph above connected? Yes!

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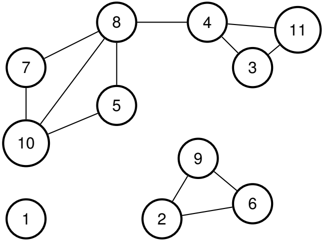
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.



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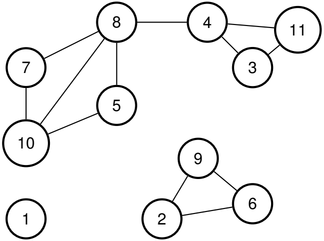

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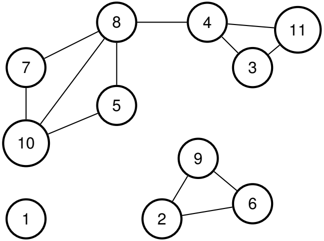

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Finally..back to Euler!

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Tour enters and leaves vertex v on each visit.

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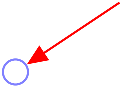

When you enter,

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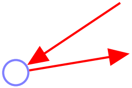

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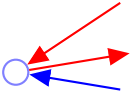

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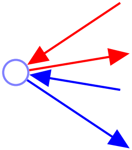

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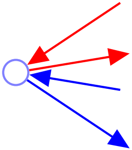

When you enter, you leave.
For starting node,

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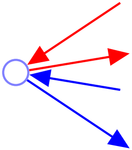

When you enter, you leave.
For starting node, tour leaves first

Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Proof of only if: Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
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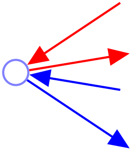

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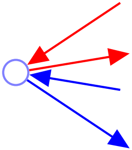

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Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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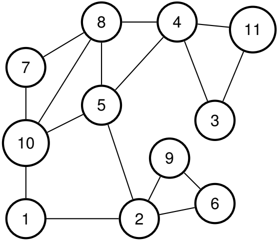

1. Take a walk starting from v (1) on “unused” edges

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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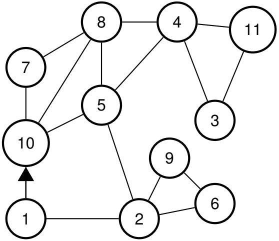

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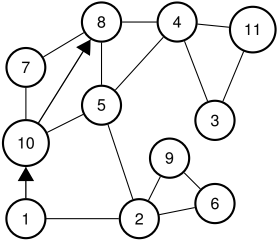

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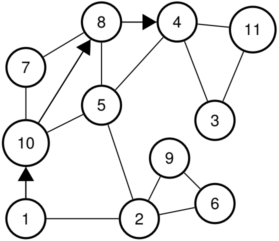

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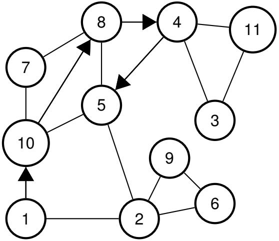

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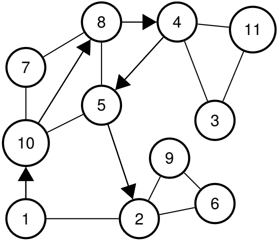

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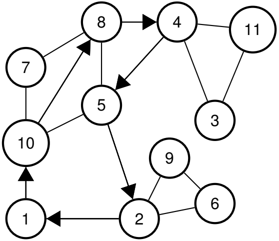

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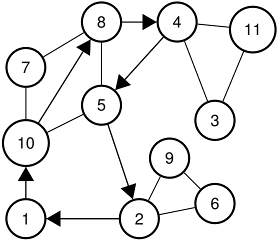

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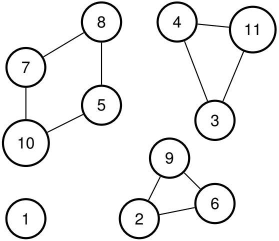

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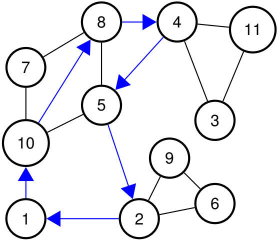

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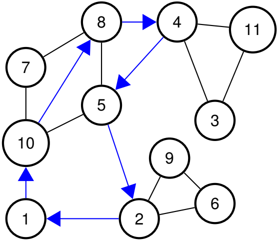

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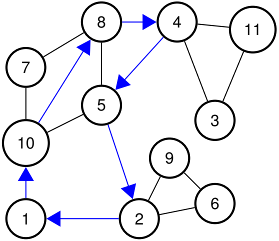

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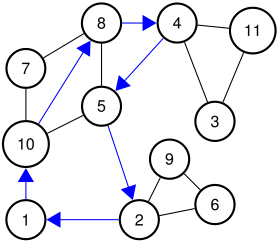

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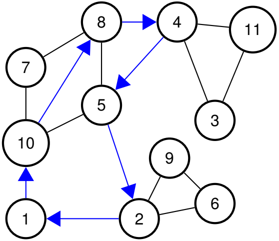

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    Example: $v_1 = 1$,

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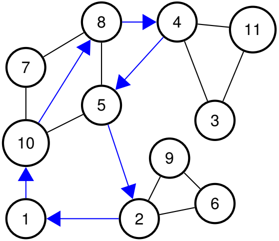

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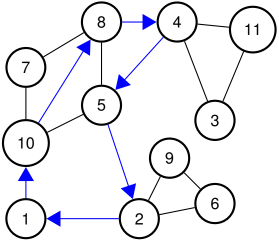

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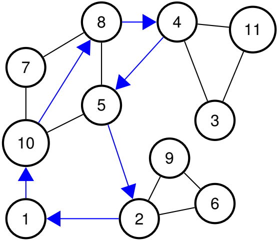

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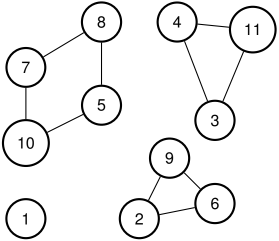

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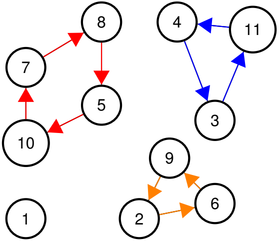

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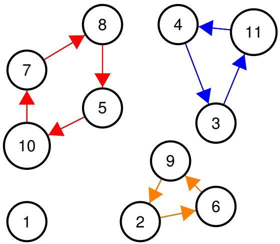

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5. Splice together.

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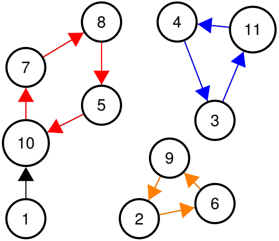

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   1,10

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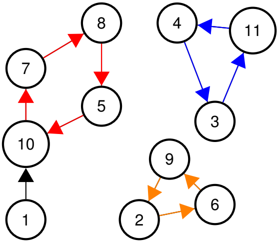

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   1,10,7,8,5,10

Finding a tour!

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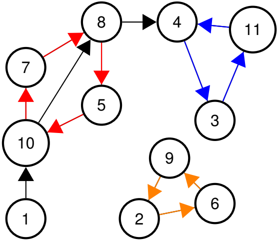

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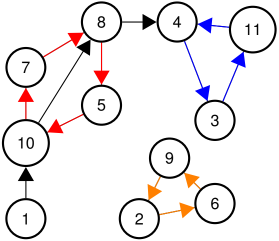

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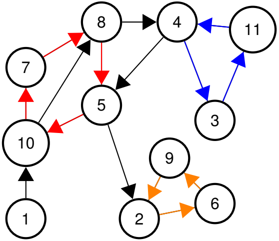

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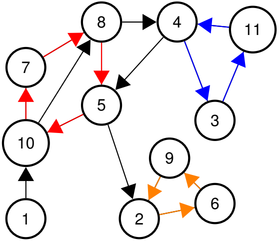

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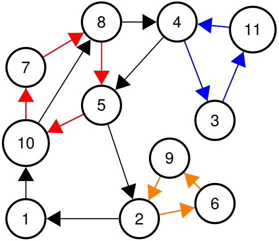

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Proof of Claim: Even degree.

Recursive/Inductive Algorithm.

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Proof of Claim: Even degree. If enter, can leave

Recursive/Inductive Algorithm.

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Proof of Claim: Even degree. If enter, can leave except for v.

Recursive/Inductive Algorithm.

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2. Remove cycle, C, from G.

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Resulting graph may be disconnected. (Removed edges!)

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Why is there a v_i in C?

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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G was connected $\Longrightarrow$

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

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3. Find tour T_i of G_i starting/ending at v_i.

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3. Find tour T_i of G_i starting/ending at v_i. Induction.

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Visits every edge once:
Visits edges in C

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Visits edges in C exactly once.

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By induction for all edges in each G_i.

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Break time!

Well admin time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

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Test Option: don't have to do homework.

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Test Option: don't have to do homework. Yes!!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework. Yes!!

Should do homework.

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Test Option: don't have to do homework. Yes!!

Should do homework. No need to write up.

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The truth:

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The truth: mostly test,

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Variance mostly in exams for A/B range.

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How will I do?

Break time!

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How will I do?

Mostly up to you.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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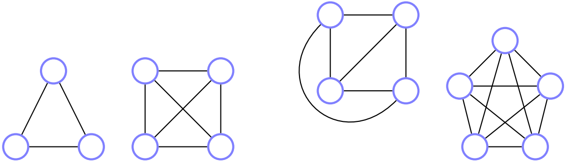

Planar?

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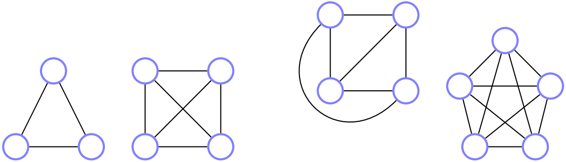

Planar? Yes for Triangle.

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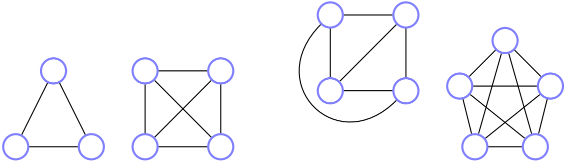

Planar? Yes for Triangle.
Four node complete?

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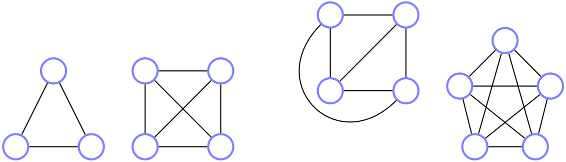

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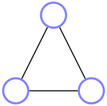 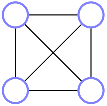 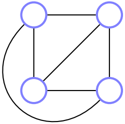 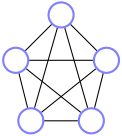

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5?

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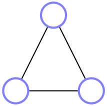 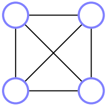 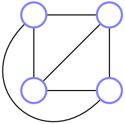 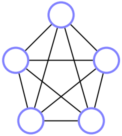

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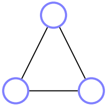
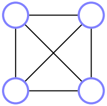
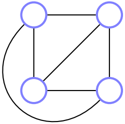
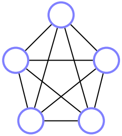

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Five node complete or K_5? No! Why?

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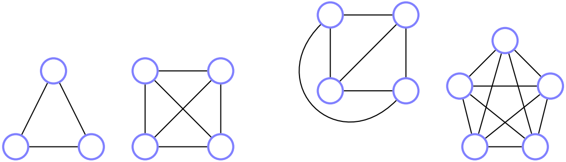

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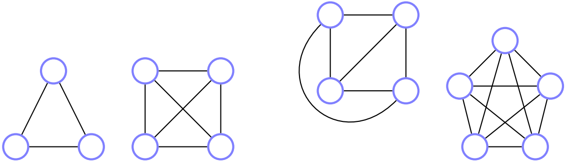

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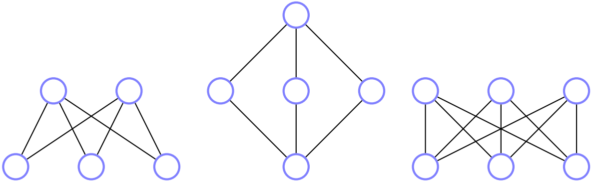

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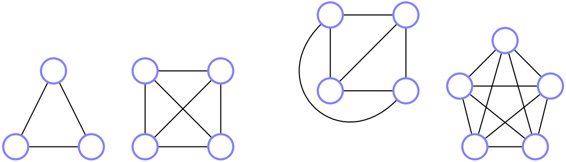

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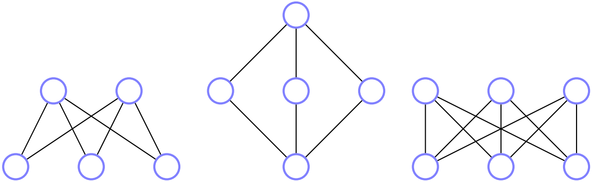

Two to three nodes, bipartite?

Planar graphs.

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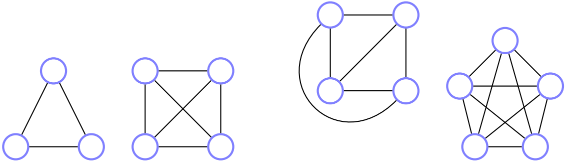

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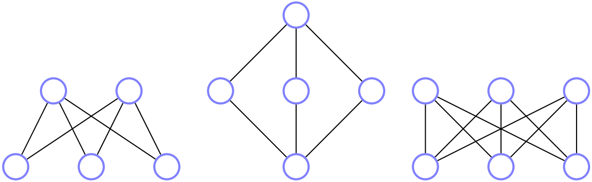

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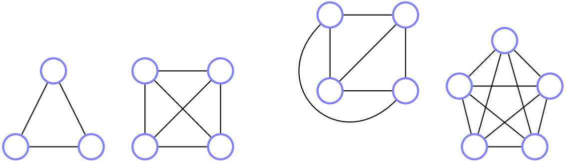

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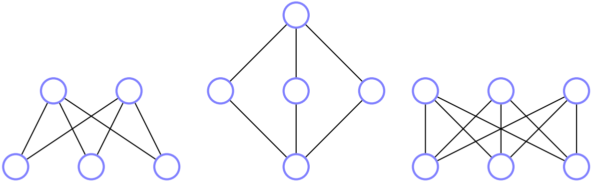

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$.

Planar graphs.

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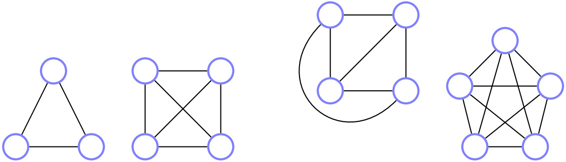

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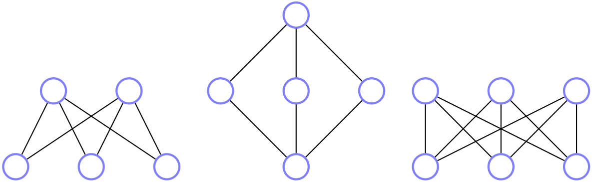

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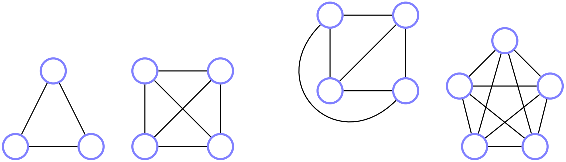

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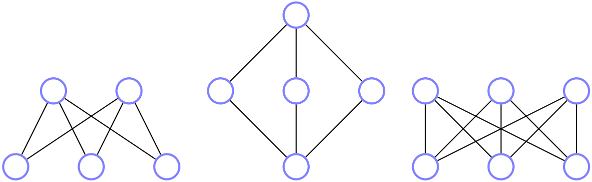

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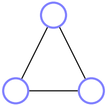
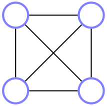
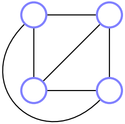
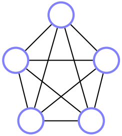
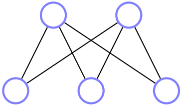
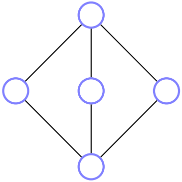
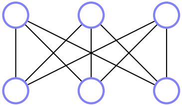
Planar graphs.

A graph that can be drawn in the plane without edge crossings.

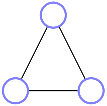
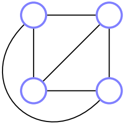
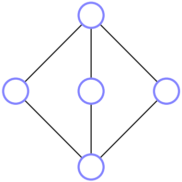





Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5? No! Why? Later.

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$. No. Why? Later.

Euler's Formula.

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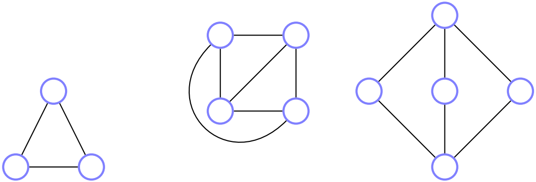
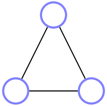
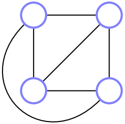
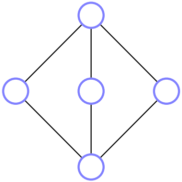


Faces: connected regions of the plane.

Euler's Formula.

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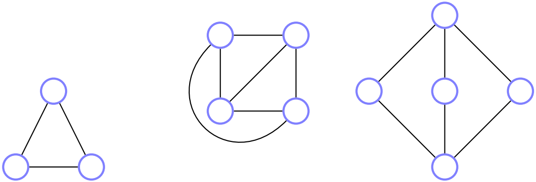
How many faces for

Euler's Formula.

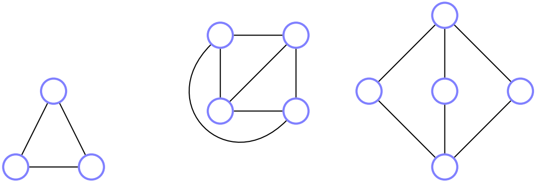


Faces: connected regions of the plane.

How many faces for

triangle?

Euler's Formula.



Faces: connected regions of the plane.

How many faces for
triangle? 2

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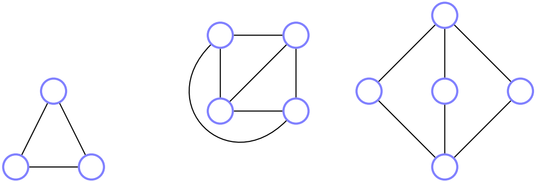


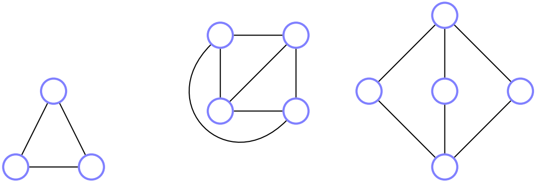
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or K_4?

Euler's Formula.



Faces: connected regions of the plane.

How many faces for
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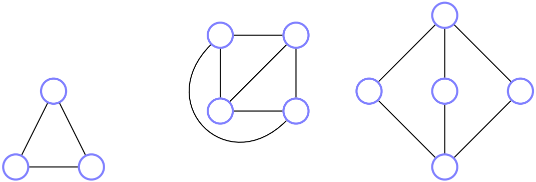

Faces: connected regions of the plane.

How many faces for

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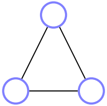
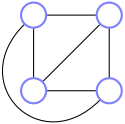
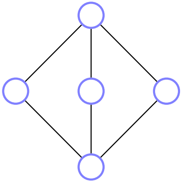
bipartite, complete two/three or $K_{2,3}$?

Euler's Formula.

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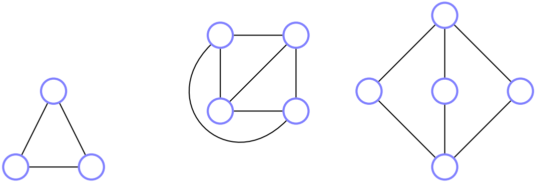

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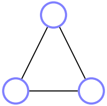

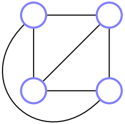

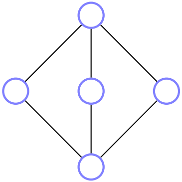

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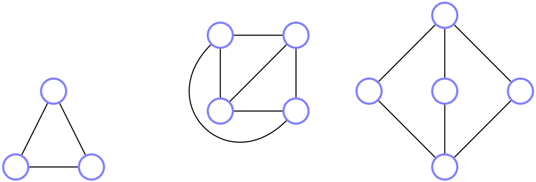

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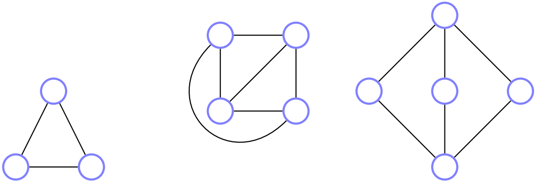

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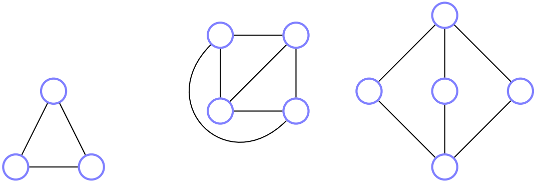

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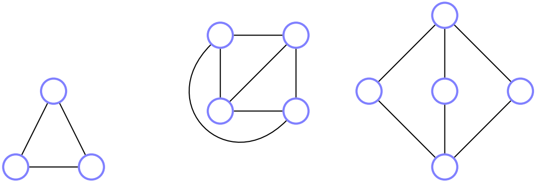

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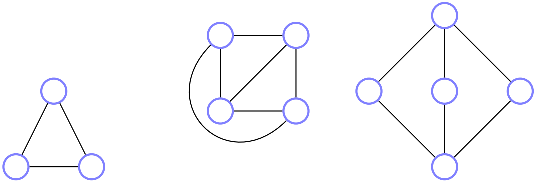

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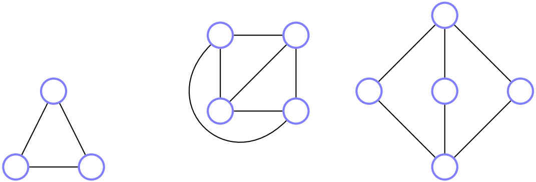
Euler's Formula.



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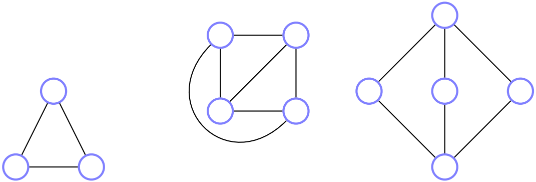
Euler's Formula.



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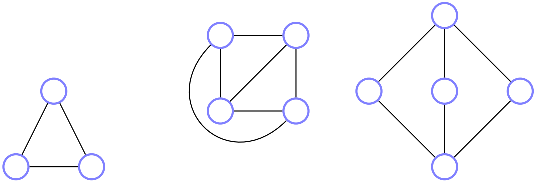

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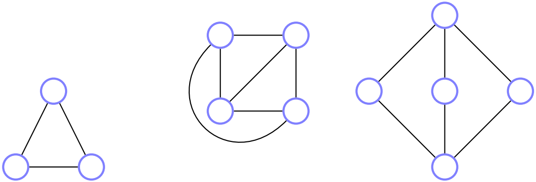

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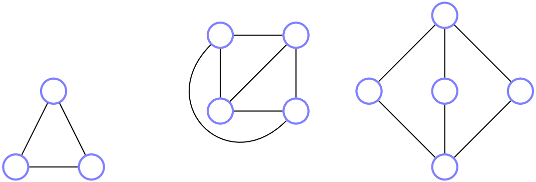

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Examples = 3!

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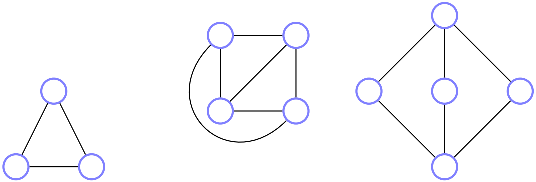

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Examples = 3! Proven!

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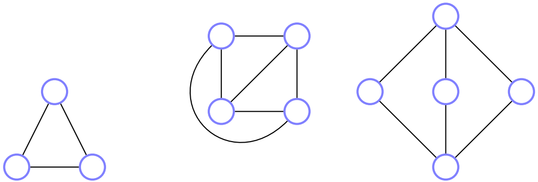

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Examples = 3! Proven! Not!!!!

Euler and Polyhedron.

Greeks knew formula for polyhedron.

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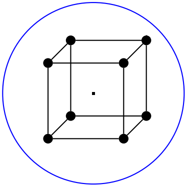

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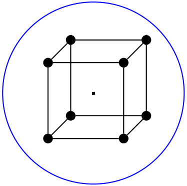

Faces?

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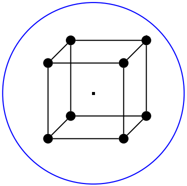

Faces? 6.   Edges?

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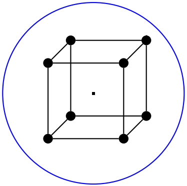

Faces? 6.   Edges? 12.

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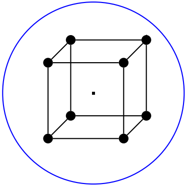

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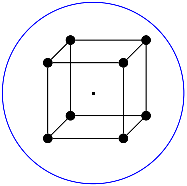

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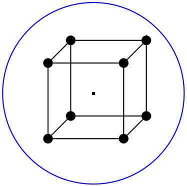

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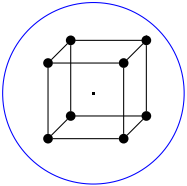

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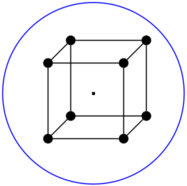

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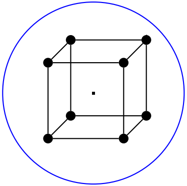

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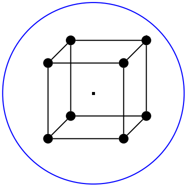

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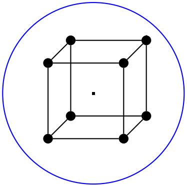

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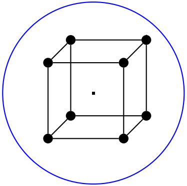

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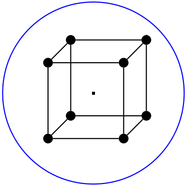

Faces? 6.   Edges? 12.   Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.
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Polyhedron without holes

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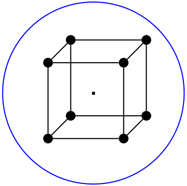

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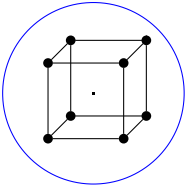

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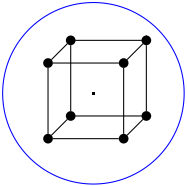

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Surround by sphere.

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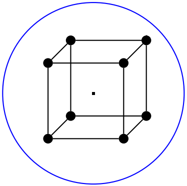

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Project from point inside polytope onto sphere.

Euler and Polyhedron.

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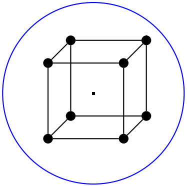

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Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

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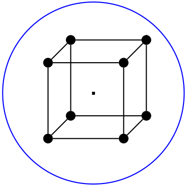

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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

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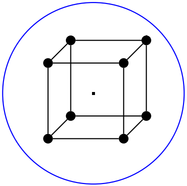

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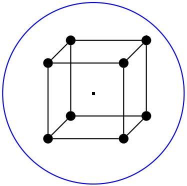

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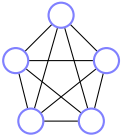
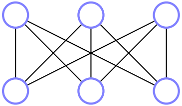
Euler proved formula thousands of years later!

Euler and planarity of K_5 and $K_{3,3}$





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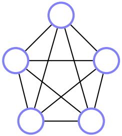

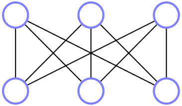

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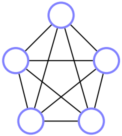

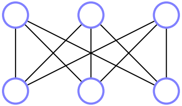

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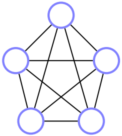

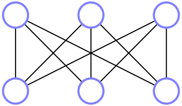

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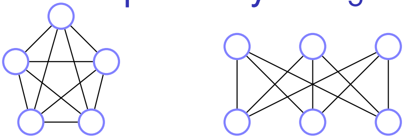

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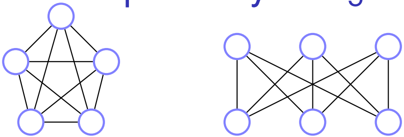

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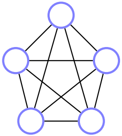

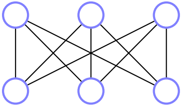

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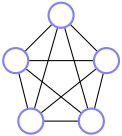

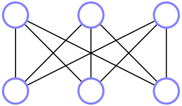

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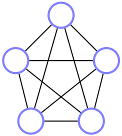

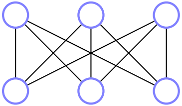

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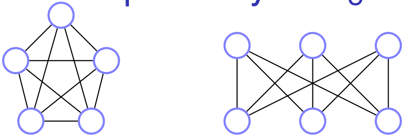

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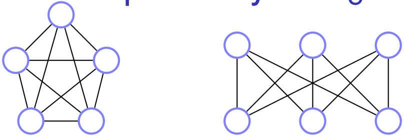

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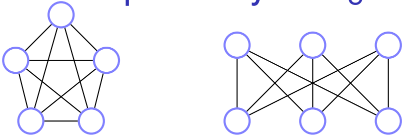

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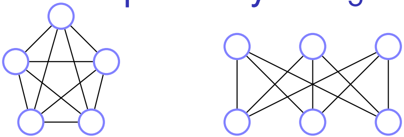

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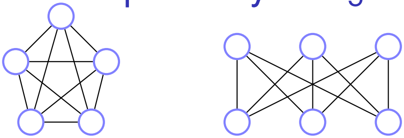
Euler and planarity of K_5 and $K_{3,3}$



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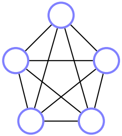
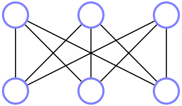
for graphs with every edge on a cycle.

Euler and planarity of K_5 and $K_{3,3}$





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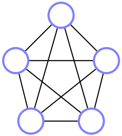
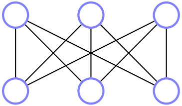
K_5

Euler and planarity of K_5 and $K_{3,3}$





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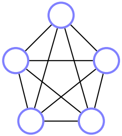
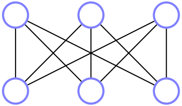
K_5 Edges?

Euler and planarity of K_5 and $K_{3,3}$





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K_5 Edges? $4 + 3 + 2 + 1$

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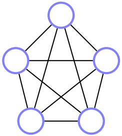

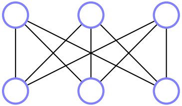

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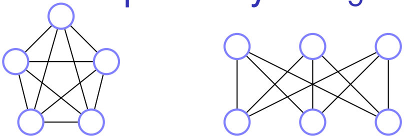
K_5 Edges? $4+3+2+1=10$.

Euler and planarity of K_5 and $K_{3,3}$



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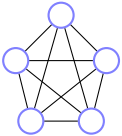
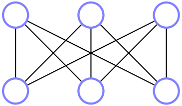
K_5 Edges? $4+3+2+1 = 10$. Vertices?

Euler and planarity of K_5 and $K_{3,3}$





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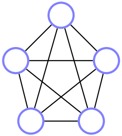
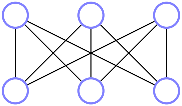
K_5 Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.

Euler and planarity of K_5 and $K_{3,3}$





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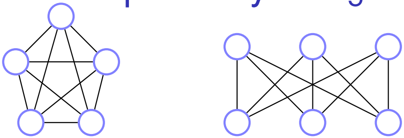

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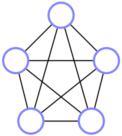

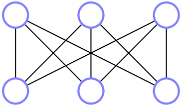

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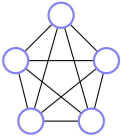

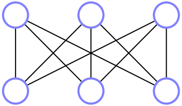

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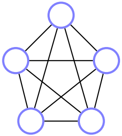

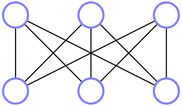

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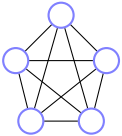

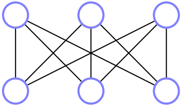

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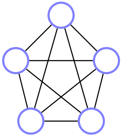

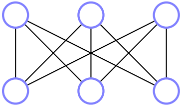

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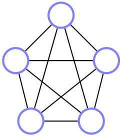

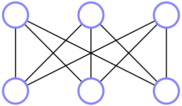

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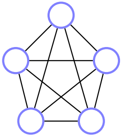

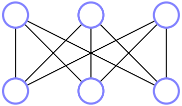

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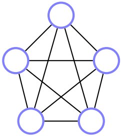

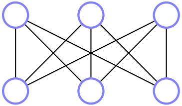

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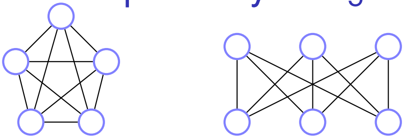

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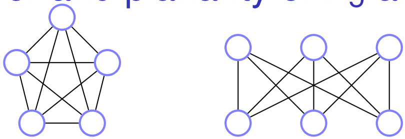

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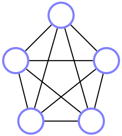

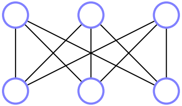

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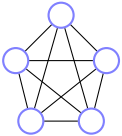

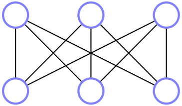

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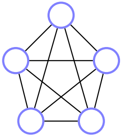

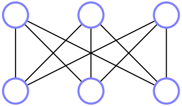

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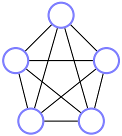

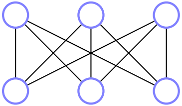

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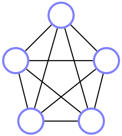

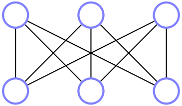

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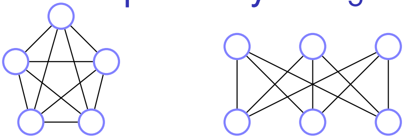

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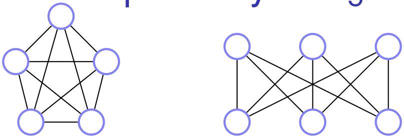

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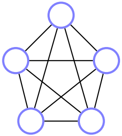
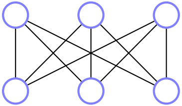
Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

Euler and planarity of K_5 and $K_{3,3}$





Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to at least three edges.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ for any planar graph.

Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

for graphs with every edge on a cycle.

K_5 Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.

$10 \not\leq 3(5) - 6 = 9$. $\implies K_5$ is not planar.

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

But no cycles that are triangles. Face is of length ≥ 4.

.... $4f \leq 2e$ for any bipartite planar graph.

Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

Tree.

A tree is a connected acyclic graph.

Tree.

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To tree or not to tree!

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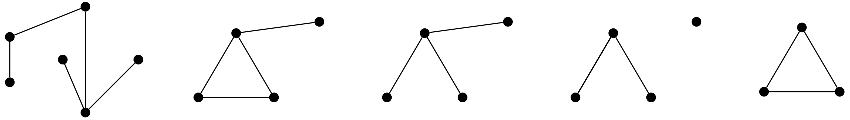

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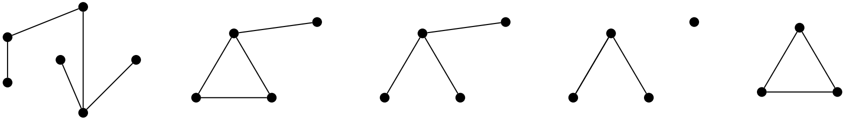

Yes.

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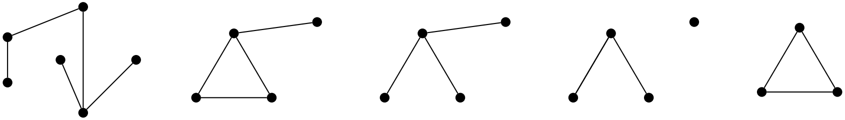

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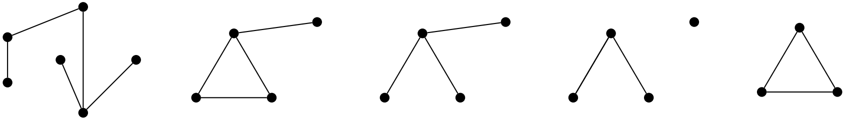

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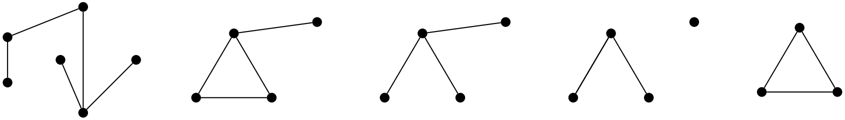

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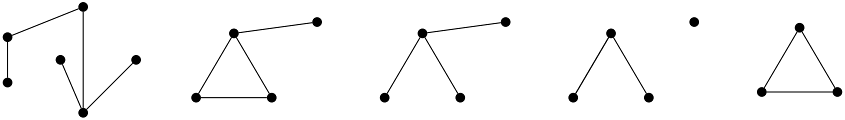

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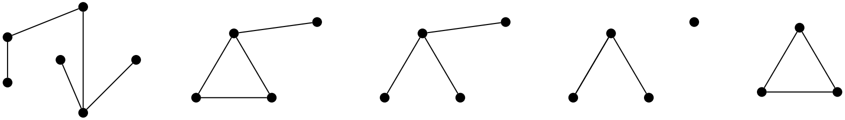

Yes. No. Yes. No. No.

Faces?

Tree.

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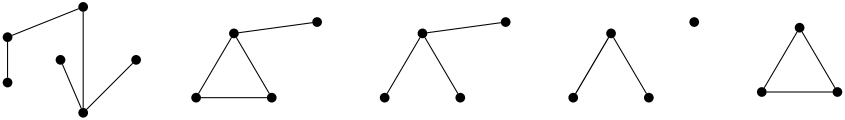

Yes. No. Yes. No. No.

Faces? 1.

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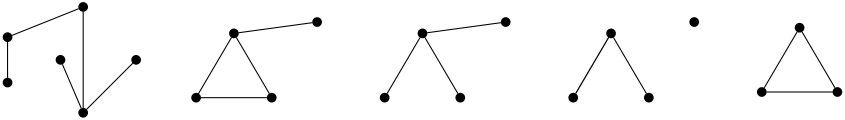

Yes. No. Yes. No. No.

Faces? 1. 2.

Tree.

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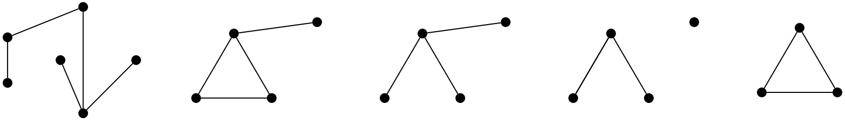

Yes. No. Yes. No. No.

Faces? 1. 2. 1.

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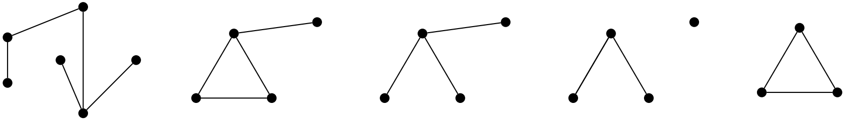

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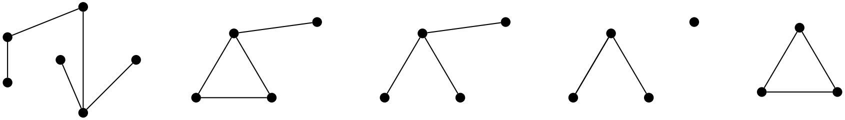

Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.

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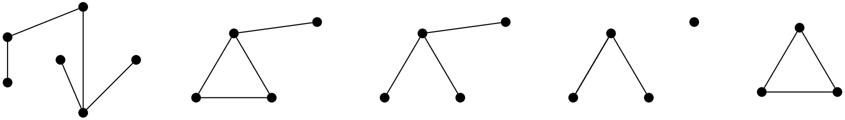

Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges.

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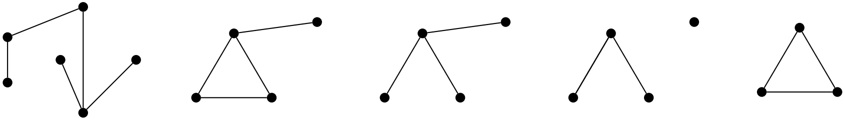

Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Notice: $e = v - 1$ for tree.

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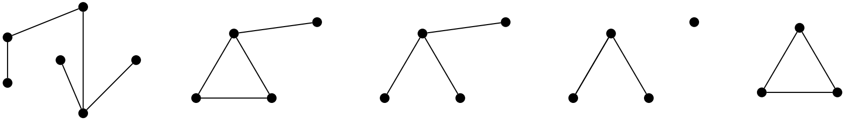

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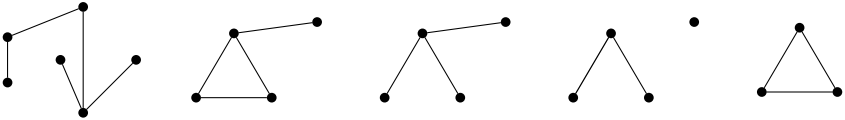

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One face for trees!

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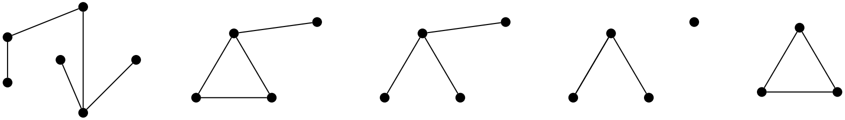

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Euler works for trees: $v + f = e + 2$.

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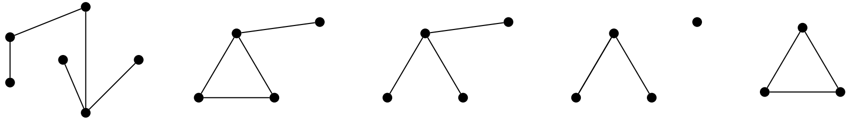

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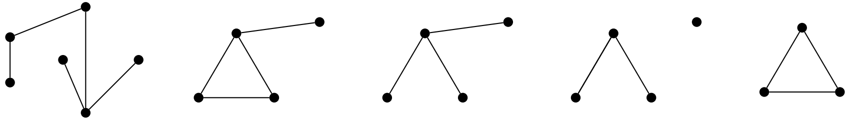

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Euler works for trees: $v + f = e + 2$.
$v + 1 = v - 1 + 2$

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on *e*.
Base:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$,

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
  If it is a tree. Done.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle.

Euler's formula.

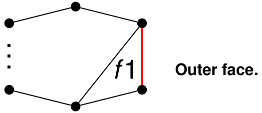
Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

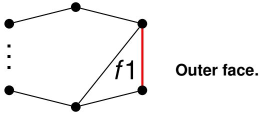


Joins two faces.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
  If it is a tree. Done.
  If not a tree.
    Find a cycle. Remove edge.

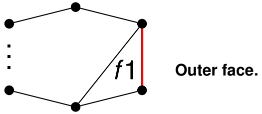


    Joins two faces.
    New graph: v-vertices.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

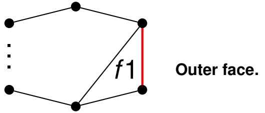


Joins two faces.
New graph: v-vertices. $e - 1$ edges.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.



Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces.

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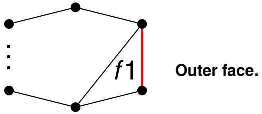

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.

Euler's formula.

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Base: $e = 0$, $v = f = 1$.
Induction Step:
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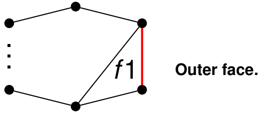

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.
$v + (f - 1) = (e - 1) + 2$ by induction hypothesis.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:
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If not a tree.
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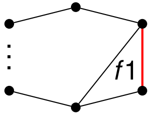

Outer face.

Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v+(f-1)=(e-1)+2$ by induction hypothesis.
Therefore $v+f=e+2$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
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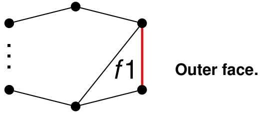

Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v + (f-1) = (e-1) + 2$ by induction hypothesis.
Therefore $v + f = e + 2$. □

Oh my goodness..what have we done!

Graphs.

Oh my goodness..what have we done!

Graphs.
  Basics.

Oh my goodness..what have we done!

Graphs.
  Basics.
Connectivity.

Oh my goodness..what have we done!

Graphs.
 Basics.
Connectivity.
Algorithm for Eulerian Tour.

Oh my goodness..what have we done!

Graphs.
  Basics.
Connectivity.
Algorithm for Eulerian Tour.

Oh my goodness..what have we done!

Graphs.
  Basics.
Connectivity.
Algorithm for Eulerian Tour.

Also Euler's formula.

Oh my goodness..what have we done!

Graphs.
  Basics.
Connectivity.
Algorithm for Eulerian Tour.

Also Euler's formula.

Yay!