CS70: Lecture 27

1. Review: Continuous Probability
2. Bayes' Rule with Continuous RVs
3. Normal Distribution
4. Central Limit Theorem
5. Confidence Intervals
6. Wrapup.

Continuous Probability

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6. Variance of Sum of Independent RVs: If X_n are pairwise independent, $var[X_1 + \cdots + X_n] = var[X_1] + \cdots + var[X_n]$

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We used $Pr[Z \in [x, x+\delta]] \approx f_Z(x)\delta$ and given A one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3\exp\{-3x\}$.

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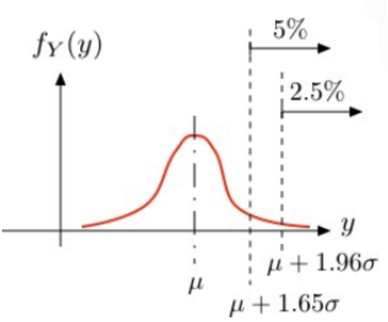

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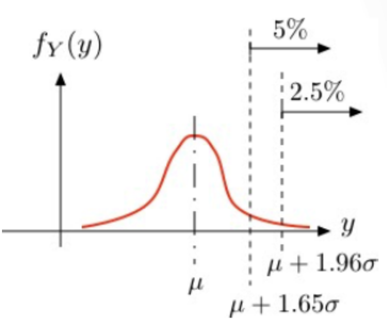

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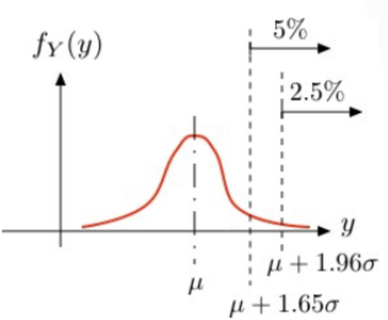

Note: $Pr[|Y - \mu| > 1.65\sigma] = 10\%$; $Pr[|Y - \mu| > 2\sigma] = 5\%$.

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Theorem Let $X = \mathcal{N}(0,1)$ and $Y = \mu + \sigma X$. Then

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Thus, the CLT provides a smaller confidence interval.

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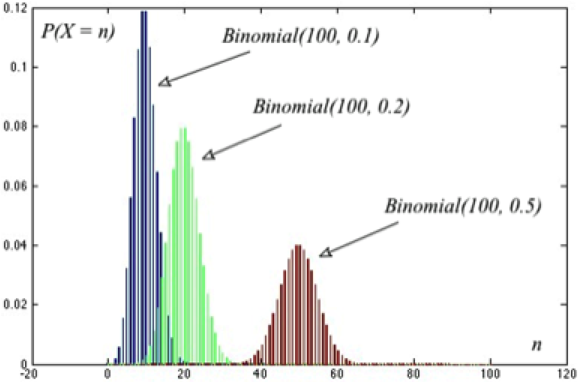

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Side note: average high school GPA is higher for female students.

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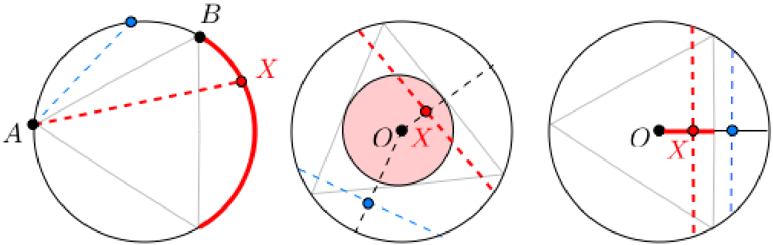
- Beware of statistics reported in the media!

Choosing at Random: Bertrand’s Paradox



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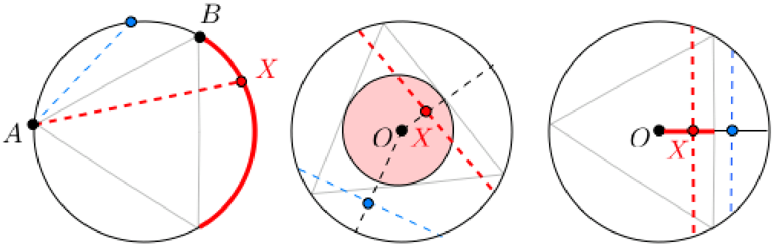

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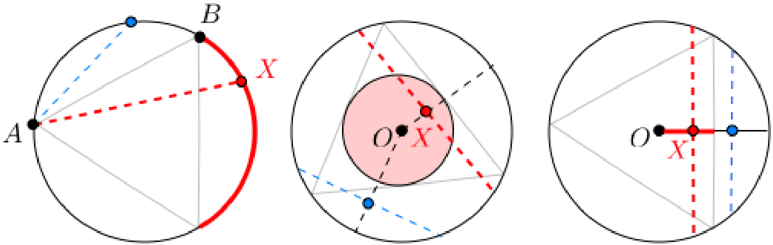

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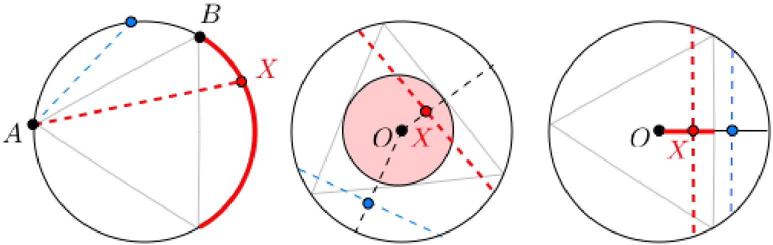

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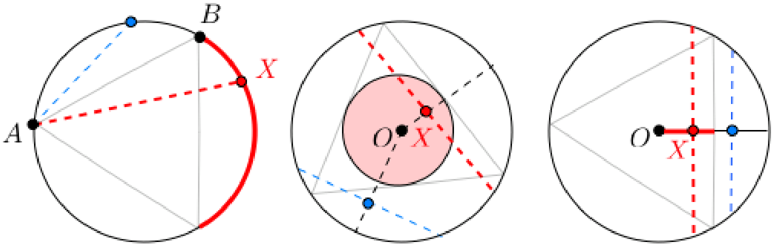

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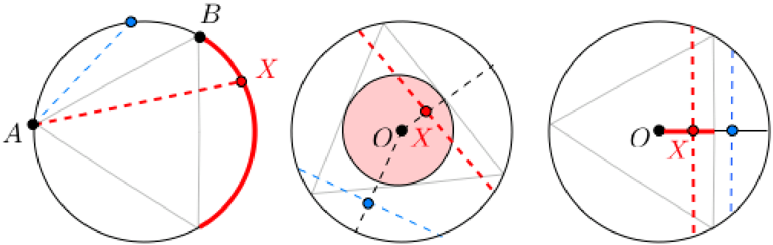

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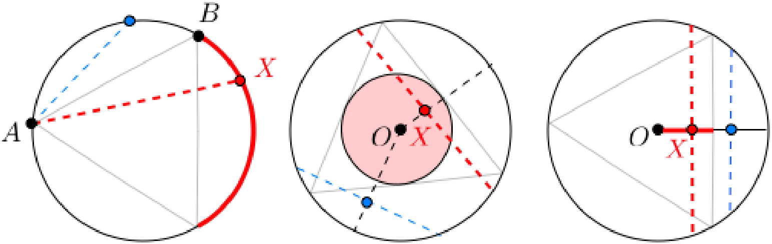

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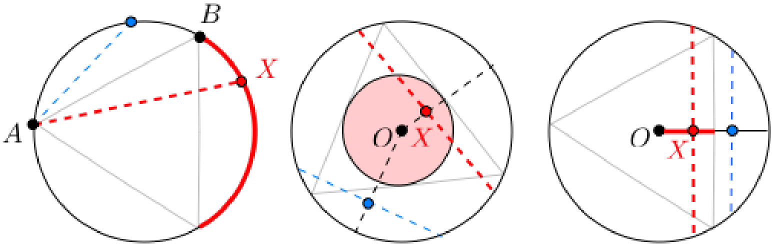

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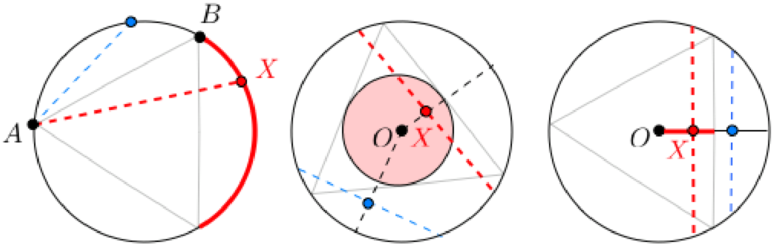

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  E.g., remember facts that confirm beliefs and forget others.

Confirmation Bias: An experiment

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As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

Surprisingly, people tend to be reinforced in their original belief, even when the evidence accumulates against it.

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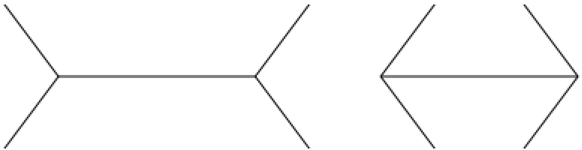

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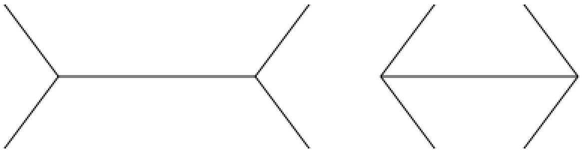

It is difficult to think clearly!

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What's Next?

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I want to learn more about discrete math and probability!

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- EE223: Stochastic Control.

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Final Thoughts

Final Thoughts

More precisely:

Final Thoughts

More precisely: Some thoughts about the final ....

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours,

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours,

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours, Reviews by TAs

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours, Reviews by TAs
- Next week: reviews during normal lecture hours:

Final Thoughts

More precisely: Some thoughts about the final ....

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- Next week: reviews during normal lecture hours:
  - Concept Review (Tuesday);

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

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- TA Office Hours, Prof. Office Hours, Reviews by TAs
- Next week: reviews during normal lecture hours:
  - Concept Review (Tuesday);
  - Question Review (Thursday).

Parting Thoughts

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You have learned a lot in this course!

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Proofs,

Parting Thoughts

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Proofs, Graphs,

Parting Thoughts

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Proofs, Graphs, Mod(p),

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Proofs, Graphs, Mod(p), RSA,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,

Parting Thoughts

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,

how to handle stress,

Parting Thoughts

You have learned a lot in this course!

Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less,

Parting Thoughts

You have learned a lot in this course!

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how to handle stress, how to sleep less, how to keep smiling, ...

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Difficult course?

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Difficult course? Yes!

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Useful?

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Finally,

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Thanks for taking the course!

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Thanks for taking the course!

Thanks to the CS70 Staff:

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- The Terrific Tutors
- The Rigorous Readers
- The Thrilling TAs
- The Amazing Assistants

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See you on Tuesday.