CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$,

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No! No q where true.

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No! No q where true.

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq'$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b-c = aq - aq' = a(q-q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ $\square$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ □

Works for $\forall a, b, c$?

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ □

Works for $\forall a,b,c$?
Argument applies to *every* $a,b,c \in Z$.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ □

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$

$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?

Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:

Goal: $P \implies Q$

Assume P.

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a,b,c$?
Argument applies to *every* $a,b,c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$
Assume P.
$\ldots$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$

$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?

Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:

Goal: $P \implies Q$

Assume P.

...

Therefore Q.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is 605

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$,

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum:

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n,

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. □

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. $\square$

Direct proof of $P \implies Q$:
Assumed P: $11|a - b + c$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1-2+1=0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6-0+5=11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a+10b+c$, for some a, b, c.

Assume: Alt. sum: $a-b+c = 11k$ for some integer k.

Add $99a+11b$ to both sides.

$$100a+10b+c = 11k+99a+11b = 11(k+9a+b)$$

Left hand side is n, $k+9a+b$ is integer. $\implies 11|n$. □

Direct proof of $P \implies Q$:
Assumed P: $11|a-b+c$. Proved Q: $11|n$.

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

Is converse a theorem?
$\forall n \in D_3, (11 | n) \implies (11 | \text{alt. sum of digits of } n)$

The Converse

Thm: $\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$

Is converse a theorem?

$\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Yes?

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

Is converse a theorem?

$\forall n \in D_3, (11 | n) \implies (11 | \text{alt. sum of digits of } n)$

Yes? No?

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof:

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$n = 100a + 10b + c = 11k$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$n = 100a + 10b + c = 11k \implies$
$99a + 11b + (a - b + c) = 11k$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k \implies \\
99a + 11b + (a - b + c) = 11k \implies \\
a - b + c = 11k - 99a - 11b
\end{aligned}
$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &
\end{aligned}
$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Proof: Assume $11|n$.

$$\begin{aligned} n = 100a + 10b + c = 11k &\implies \\ 99a + 11b + (a - b + c) = 11k &\implies \\ a - b + c = 11k - 99a - 11b &\implies \\ a - b + c = 11(k - 9a - b) &\implies \\ a - b + c = 11\ell & \end{aligned}$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &\implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) &\in Z
\end{aligned}
$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &\implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z &
\end{aligned}
$$

That is 11|alternating sum of digits. □

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$\begin{aligned} n = 100a + 10b + c = 11k \implies \\ 99a + 11b + (a - b + c) = 11k \implies \\ a - b + c = 11k - 99a - 11b \implies \\ a - b + c = 11(k - 9a - b) \implies \\ a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z \end{aligned}$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &\implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z &
\end{aligned}
$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$\begin{aligned} n = 100a + 10b + c = 11k \implies \\ 99a + 11b + (a - b + c) = 11k \implies \\ a - b + c = 11k - 99a - 11b \implies \\ a - b + c = 11(k - 9a - b) \implies \\ a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z \end{aligned}$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k \implies \\
99a + 11b + (a - b + c) = 11k \implies \\
a - b + c = 11k - 99a - 11b \implies \\
a - b + c = 11(k - 9a - b) \implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z
\end{aligned}
$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

We have.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k \implies$$
$$99a + 11b + (a - b + c) = 11k \implies$$
$$a - b + c = 11k - 99a - 11b \implies$$
$$a - b + c = 11(k - 9a - b) \implies$$
$$a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

We have.

Theorem: $\forall n \in N', (11|\text{alt. sum of digits of } n) \iff (11|n)$

Proof by Contraposition

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d | n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k)$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

n is even.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

n is even. $\neg P$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

n is even. $\neg P$ $\square$

Another Contraposition...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

n^2 is even, $n^2 = 2k$, ...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

n^2 is even, $n^2 = 2k$, ...$\sqrt{2k}$ even?

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' ....

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' ....

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$ so $P \implies Q$ and ...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$ so $P \implies Q$ and ... $\square$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show:

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$,

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

Contrapositive: True $\implies P$.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

Contrapositive: True $\implies P$. Theorem P is proven.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

Contrapositive: True $\implies P$. Theorem P is proven. $\square$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$:

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

$$\sqrt{2}b = a$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

a^2 is even $\implies$ a is even.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and *b* have a common factor.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: *a* and *b* have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and *b* have a common factor. Contradiction.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and b have a common factor. Contradiction.

□

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q,

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.
- so $p \leq 1$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.
- so $p \leq 1$. (**Contradicts** R**.**)

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.
- so $p \leq 1$. (**Contradicts *R*.**)

The original assumption that "the theorem is false" is false, thus the theorem is proven.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.
- so $p \leq 1$. (**Contradicts *R*.**)

The original assumption that "the theorem is false" is false, thus the theorem is proven. □

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$
- There is a prime *in between* 13 and $q = 30031$ that divides q.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$
- There is a prime *in between* 13 and $q = 30031$ that divides q.
- Proof assumed no primes *in between* p_k and q.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

Proof: First a lemma...

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even!

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. $\square$

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. $\square$

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even. Not possible.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even. Not possible.
Case 4: a even, b even: even - even +even = even.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. $\square$

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even. Not possible.
Case 4: a even, b even: even - even +even = even. Possible.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even. Not possible.
Case 4: a even, b even: even - even +even = even. Possible.

The fourth case is the only one possible,

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even. Not possible.
Case 4: a even, b even: even - even +even = even. Possible.

The fourth case is the only one possible, so the lemma follows. □

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
-

$$x^y =$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
-

$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}}$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} * \sqrt{2}} = \sqrt{2}^2$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^2 = 2.$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^{2} = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^{2} = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^{2} = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds. □

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
-
$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^2 = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds. □

Question: Which case holds?

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^2 = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds. □

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get

$$4 = 3.$$

By commutativity

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get

$$4 = 3.$$

By commutativity theorem holds.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$$4 = 3.$$

By commutativity theorem holds. $\square$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get

$$4 = 3.$$

By commutativity theorem holds. $\square$

Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
Proof:

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$
$$x(x - y) = (x + y)(x - y)$$

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x-y) &= (x+y)(x-y) \\
x &= (x+y)
\end{aligned}
$$

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \\ x &= 2x \end{aligned}$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x-y) &= (x+y)(x-y) \\
x &= (x+y) \\
x &= 2x \\
1 &= 2
\end{aligned}
$$

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \\ x &= 2x \\ 1 &= 2 \end{aligned}$$

□

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x-y) &= (x+y)(x-y) \\
x &= (x+y) \\
x &= 2x \\
1 &= 2
\end{aligned}
$$

□

Dividing by zero is no good.

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x - y) &= (x + y)(x - y) \\ x &= (x + y) \\ x &= 2x \\ 1 &= 2 \end{aligned}$$

□

Dividing by zero is no good.

Also: Multiplying inequalities by a negative.

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \\ x &= 2x \\ 1 &= 2\end{aligned}$$

□

Dividing by zero is no good.

Also: Multiplying inequalities by a negative.

$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

Direct Proof:

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!
Don't assume the theorem.

Summary: Note 2.

Direct Proof:
 To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
 To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
 To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
 Universal: show that statement holds in all cases.
 Existence: used cases where one is true.
  Either $\sqrt{2}$ and $\sqrt{2}$ worked.
   or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!
  Don't assume the theorem. Divide by zero.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!
Don't assume the theorem. Divide by zero.Watch converse.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

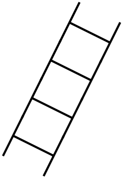
Careful when proving!
Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

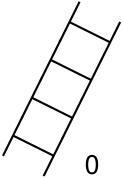
The natural numbers.

The natural numbers.



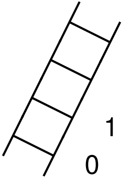
The natural numbers.

0,



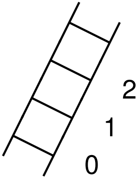
The natural numbers.

0, 1,



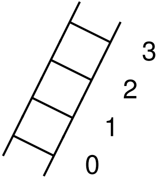
The natural numbers.

0, 1, 2,

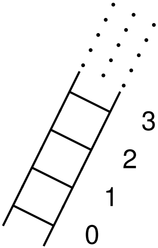


The natural numbers.

0, 1, 2, 3,



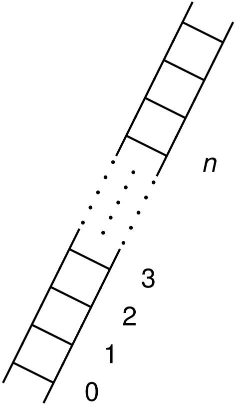
The natural numbers.



0, 1, 2, 3,

. . . ,

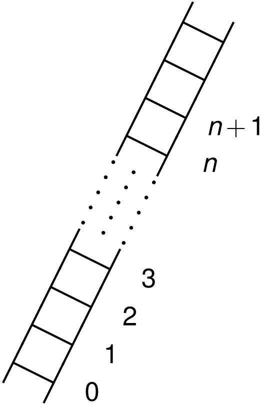
The natural numbers.



0, 1, 2, 3,

…, n,

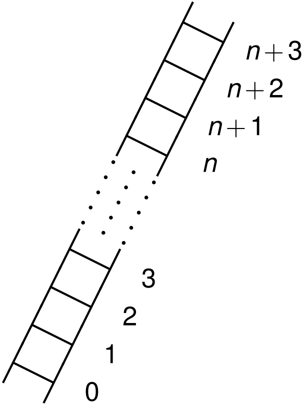
The natural numbers.



0, 1, 2, 3,

$\ldots$, n, $n+1$,

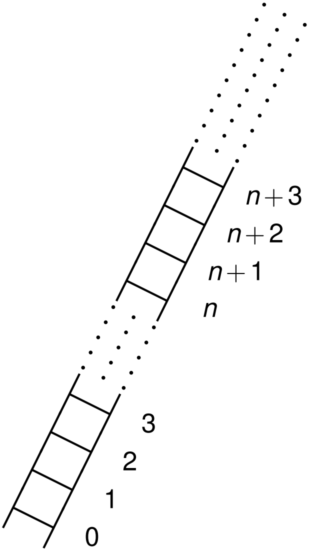
The natural numbers.



0, 1, 2, 3,

$\ldots, n, n+1, n+2, n+3,$

The natural numbers.



$0, 1, 2, 3,$

$\ldots, n, n+1, n+2, n+3, \ \ldots$

A formula.

A formula.

Teacher: Hello class.

A formula.

Teacher: Hello class.
Teacher:

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall (n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

$\forall(n \in N) : P(n)$.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

$$\forall(n \in N) : P(n).$$

$P(n)$ is "$\sum_{i=0}^{n} i \frac{(n)(n+1)}{2}$".

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

$\forall(n \in N) : P(n)$.

$P(n)$ is "$\sum_{i=0}^{n} i \frac{(n)(n+1)}{2}$".

Principle of Induction:

- Prove $P(0)$.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

$\forall(n \in N) : P(n)$.

$P(n)$ is "$\sum_{i=0}^{n} i \frac{(n)(n+1)}{2}$".

Principle of Induction:

- Prove $P(0)$.
- Assume $P(k)$, "Induction Hypothesis"

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Five year old Gauss Theorem: $\forall(n \in N) : \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$.

It is a statement about all natural numbers.

$\forall(n \in N) : P(n)$.

$P(n)$ is "$\sum_{i=0}^{n} i \frac{(n)(n+1)}{2}$".

Principle of Induction:

- Prove $P(0)$.
- Assume $P(k)$, "Induction Hypothesis"
- Prove $P(k+1)$. "Induction Step."

Gauss induction proof.

Theorem: For all natural numbers n, $0 + 1 + 2 \cdots n = \frac{n(n+1)}{2}$

Gauss induction proof.

Theorem: For all natural numbers n, $0 + 1 + 2 \cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$?

Gauss induction proof.

Theorem: For all natural numbers n, $0 + 1 + 2 \cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$1+\cdots+k+(k+1) \quad = \quad \frac{k(k+1)}{2}+(k+1)$$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$\begin{aligned} 1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\ &= \frac{k^2+k+2(k+1)}{2} \end{aligned}$$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$
\begin{aligned}
1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\
&= \frac{k^2+k+2(k+1)}{2} \\
&= \frac{k^2+3k+2}{2}
\end{aligned}
$$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$\begin{aligned}
1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\
&= \frac{k^2+k+2(k+1)}{2} \\
&= \frac{k^2+3k+2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}$$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$\begin{aligned} 1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\ &= \frac{k^2+k+2(k+1)}{2} \\ &= \frac{k^2+3k+2}{2} \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$\begin{aligned}
1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\
&= \frac{k^2+k+2(k+1)}{2} \\
&= \frac{k^2+3k+2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}$$

$P(k+1)$!.

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$
\begin{aligned}
1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\
&= \frac{k^2+k+2(k+1)}{2} \\
&= \frac{k^2+3k+2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}
$$

$P(k+1)$!. By principle of induction...

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

Base Case: Does 0 = $\frac{0(0+1)}{2}$? Yes.

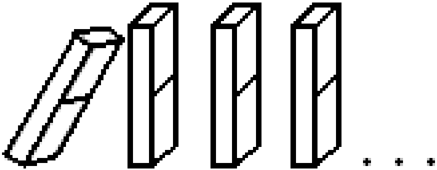
Induction Step: Show $\forall k \geq 0, P(k) \implies P(k+1)$
Induction Hypothesis: $P(k) = 1+\cdots+k = \frac{k(k+1)}{2}$

$$\begin{aligned} 1+\cdots+k+(k+1) &= \frac{k(k+1)}{2}+(k+1) \\ &= \frac{k^2+k+2(k+1)}{2} \\ &= \frac{k^2+3k+2}{2} \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

$P(k+1)$!. By principle of induction... □

Notes visualization

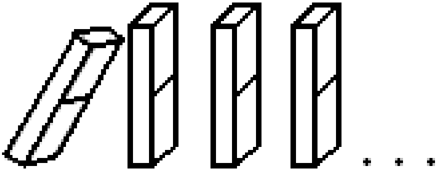
Note's visualization: an infinite sequence of dominos.



Prove they all fall down;

Notes visualization

Note's visualization: an infinite sequence of dominos.

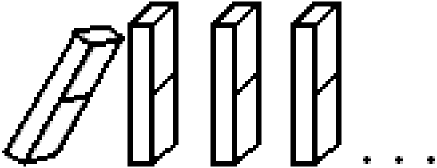


Prove they all fall down;

- $P(0)$ = "First domino falls"

Notes visualization

Note's visualization: an infinite sequence of dominos.

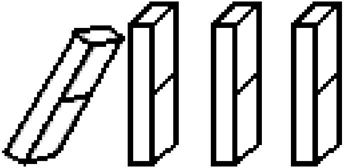


Prove they all fall down;

- $P(0)$ = "First domino falls"
- $(\forall k)\, P(k) \implies P(k+1)$:

Notes visualization

Note's visualization: an infinite sequence of dominos.

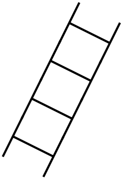
 . . .

Prove they all fall down;

- $P(0)$ = "First domino falls"
- $(\forall k)\ P(k) \implies P(k+1)$:
  "kth domino falls implies that $k+1$st domino falls"

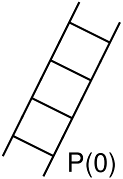
Climb an infinite ladder?

Climb an infinite ladder?



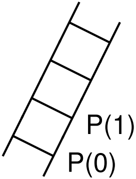
Climb an infinite ladder?

$P(0)$



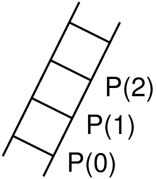
Climb an infinite ladder?

$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$



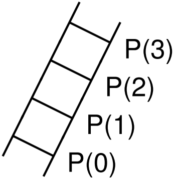
Climb an infinite ladder?

$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2)$$

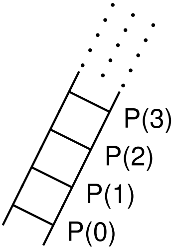


Climb an infinite ladder?

$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3)$$

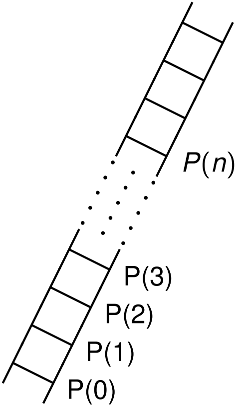


Climb an infinite ladder?



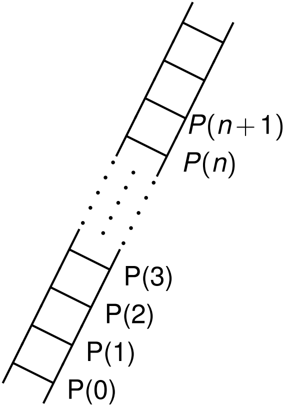
$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$

Climb an infinite ladder?



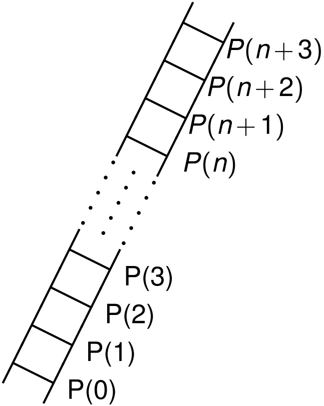
$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$

Climb an infinite ladder?



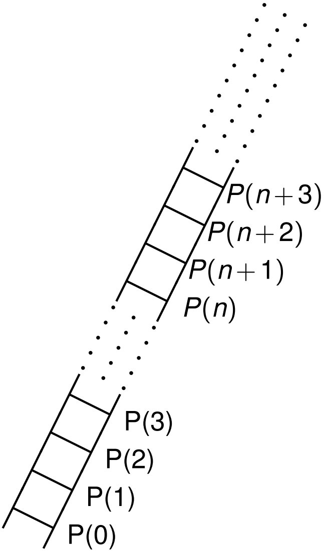
$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$

Climb an infinite ladder?



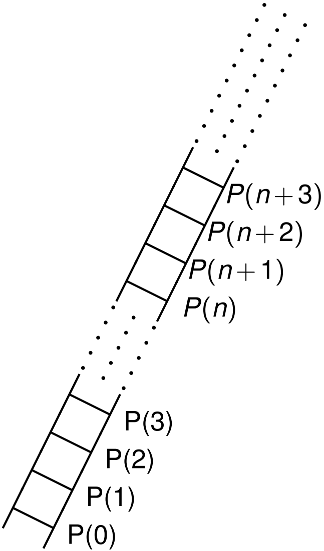
$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \dots$$

Climb an infinite ladder?



$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$
$$(\forall n \in N) P(n)$$

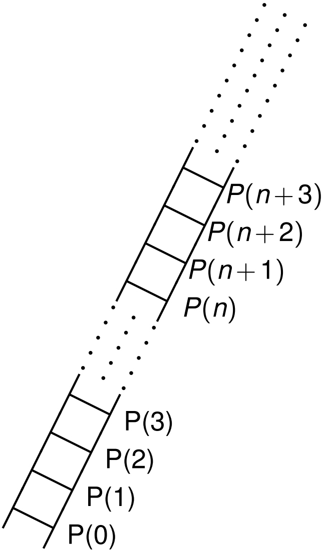
Climb an infinite ladder?



$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$
$$(\forall n \in N) P(n)$$

Your favorite example of forever..

Climb an infinite ladder?



$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$
$$P(0) \implies P(1) \implies P(2) \implies P(3) \ldots$$
$$(\forall n \in N) P(n)$$

Your favorite example of forever..or the natural numbers...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k + 1$?

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1)$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof?

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true
plus inductive step

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true
plus inductive step $\implies$ true for $n = 1$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true
plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$
plus inductive step $\implies$ true for $n = 2$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

$\ldots$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k \implies$ true for $n = k+1$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k \implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k \implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k \implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k$ $\implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

...

Predicate, $P(n)$, True for all natural numbers!

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k$ $\implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

...

Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1+2\cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"
  - Prove $P(k+1)$. "Induction Step."

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"
  - Prove $P(k+1)$. "Induction Step."

$P(n)$ true for all natural numbers n!!!

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. “Base Case”.
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, “Induction Hypothesis”
  - Prove $P(k+1)$. “Induction Step.”

$P(n)$ true for all natural numbers n!!!
Get to use $P(k)$ to prove $P(k+1)$!

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. “Base Case”.
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, “Induction Hypothesis”
  - Prove $P(k+1)$. “Induction Step.”

$P(n)$ true for all natural numbers n!!!
Get to use $P(k)$ to prove $P(k+1)$! !

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
    - Assume $P(k)$, "Induction Hypothesis"
    - Prove $P(k+1)$. "Induction Step."

$P(n)$ true for all natural numbers n!!!
Get to use $P(k)$ to prove $P(k+1)$! ! !

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1+2\cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"
  - Prove $P(k+1)$. "Induction Step."

$P(n)$ true for all natural numbers n!!!
Get to use $P(k)$ to prove $P(k+1)$! ! ! !

Next Time.

More induction!

Next Time.

More induction!
See you on Tuesday!