Next Topic: Undecidability.

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Barber paradox.

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"The barber shaves every person who does not shave themselves."

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Get around paradox?

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Get around paradox?
The barber lies.

Russell's Paradox.

Naive Set Theory: Any definable collection is a set.

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Take $x = y$.

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What type of object is a set that contain sets?

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Axioms changed.

Changing Axioms?

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See Logicomix by Doxiaidis, Papadimitriou (professor here),

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Write me a program checker!

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Check that the compiler works!

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Run P on I and check!

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How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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Theorem: There is no program HALT.

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What is he talking about?
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What is he talking about?
(A) He is confused.
(B) Fermat's Theorem.

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What is he talking about?
(A) He is confused.
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Halt and Turing.

Proof:

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Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Turing(P)

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Assumption: there is a program HALT.
There is text that "is" the program HALT.
There is text that is the program Turing.
Can run Turing on Turing!

Does Turing(Turing) halt?

Turing(Turing) halts
$\implies$ then HALTS(Turing, Turing) $=$ halts
$\implies$ Turing(Turing) loops forever.

Turing(Turing) loops forever
$\implies$ then HALTS(Turing, Turing) $\neq$ halts
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Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Another view of proof: diagonalization.

Any program is a fixed length string.

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P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Proof play by play.

Assumed HALT(P, I) existed.

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What is P?

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Turing “diagonalizes” on list of program.

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$\Longrightarrow$ HALT is not a program.

Questions?

Wow, that was easy!

Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing’s time.

No computers for Turing!

In Turing's time.

No computers.

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No computers.

Adding machines.

No computers for Turing!

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Adding machines.
e.g., Babbage (from table of logarithms) 1812.

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Concept of program as data wasn't really there.

Turing machine.

Turing machine.

A Turing machine.
– an (infinite) tape with characters

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– an (infinite) tape with characters
– be in a state, and read a character

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Universal Turing machine

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Now that's a computer!

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Turing: AI,

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

Turing and computing.

Just a mathematician?

“Wrote” a chess program.

Turing and computing.

Just a mathematician?

“Wrote” a chess program.

Simulated the program by hand to play chess.

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It won!

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Involved with computing labs through the 40s.

Church, Gödel and Turing.

Church proved an equivalent theorem. (Previously.)

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Programming languages!

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Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

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We can't get enough of building more Turing machines.

Undecidable problems.

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How?

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How? What is P?

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How? What is P? Text!!!!!!

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Can a set of notched tiles tile the infinite plane?

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Example: “ $x^n + y^n = 1$?”

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Undecidability for Diophantine set of equations
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2013. Granted Royal pardon.

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Program is text, so we can pass it to itself,
 or refer to self.

Summary: decidability.

Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

Probability

What’s to come?

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A bag contains:

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

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For now: Counting!

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