

1 Continuous Computations

Let X be a continuous random variable whose pdf is cx^3 (for some constant c) in the range $0 \leq x \leq 1$, and is 0 outside this range.

(a) Find c .

(b) Find $\Pr[1/3 \leq X \leq 2/3 \mid X \leq 1/2]$.

(c) Find $\mathbf{E}(X)$.

(d) Find $\text{var}(X)$.

2 Predicament

Three men are on a boat with cigarettes, but they have no lighter. What do they do?

3 Guess the Polynomial

Remember the mantra “ $d + 1$ points uniquely determine a degree $\leq d$ polynomial”?

Write down a polynomial (of any degree) with non-negative, integer coefficients. Your TA will then guess the exact polynomial you have written down, using only information about the polynomial at two points!

4 Which Envelope?

You have two envelopes in front of you containing cash. You know that one envelope contains twice as much money as the other envelope. You are allowed to pick one envelope and see how much cash is inside, and then based on this information, you can decide to switch envelopes or stick with the envelope you already have.

Can you come up with a strategy which will allow you to pick the envelope with more money, with probability strictly greater than $1/2$?

5 Coloring a Sphere

Consider a sphere in which exactly $1/10$ of the surface of the sphere is colored red (the rest of the sphere is blue). Prove that no matter how the blue is distributed upon the sphere, it is always possible to inscribe a cube inside the sphere so that all of its corners are blue.