CS 70 Spring 2017 | Discrete Mathematics and Probability Theory Rao | DIS 13a

1 Uniform Means

Let $X_1, X_2, \ldots, X_n$ be n independent and identically distributed uniform random variables on the interval $[0,1]$.

(a) Let $Y = \min\{X_1, X_2, \ldots, X_n\}$. Find $\mathbf{E}(Y)$. [*Hint*: Use the tail sum formula, which says the expected value of a nonnegative random variable is $\mathbf{E}(X) = \int_0^\infty \Pr(X > x)\,\mathrm{d}x$. Note that we can use the tail sum formula since $Y \geq 0$.]

(b) Let $Z = \max\{X_1, X_2, \ldots, X_n\}$. Find $\mathbf{E}(Z)$. [*Hint*: Find the CDF.]

2 Conditioning on Exponentials

Let X_i be i.i.d. Expo(λ) random variables.

(a) Compute $\mathbf{E}[Y \mid Z]$, where $Y = \max\{X_1, X_2\}$ and $Z = \min\{X_1, X_2\}$.

(b) Compute $\mathbf{E}[X_1 + X_2 \mid Z]$. [*Hint*: Use part (a).]

(c) Use part (b) to compute $\mathbf{E}[Z]$.

(d) Compute $\mathbf{E}[X_1 + X_2 \mid X_1 + X_2 + X_3]$.

3 Bayesian Darts

You play a game of darts with your friend. You are better than he is, and the distances of your darts to the center of the target are i.i.d. $U[0,1]$ whereas his are i.i.d. $U[0,2]$. To make the game fair, you agree that you will throw one dart and he will throw two darts. The dart closest to the center wins the game. What is the probability that you will win? *Note*: The distances *from the center of the board* are uniform.