CS 70	Discrete Mathematics and Probability Theory	
Spring 2017	Rao	DIS 11b

1 Uniform Probability Space

Let $\Omega = \{1,2,3,4,5,6\}$ be a uniform probability space. Let also $X(\omega)$ and $Y(\omega)$, for $\omega \in \Omega$, be the random variables defined in the table:

Table 1: All the rows in the table correspond to random variables.

ω	1	2	3	4	5	6
$X(\omega)$	0	0	1	1	2	2
$Y(\omega)$	0	2	3	5	2	0
$X^2(\omega)$						
$Y^2(\omega)$						
$XY(\omega)$						
$L[Y \mid X](\omega)$						
$E[Y \mid X](\omega)$						

(a) Fill in the blank entries of the table.

(b) Are the variables correlated or uncorrelated? Are the variables independent or dependent?

(c) Calculate $\mathbf{E}[(Y - L[Y \mid X])^2]$ and $\mathbf{E}[(Y - \mathbf{E}[Y \mid X])^2]$. Which is smaller? Is this always true?

2 Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation.

(a) If we roll a die until we see a 6, how many ones should we expect to see?

(b) If we roll a die until we see a number greater than 3, how many ones should we expect to see?

3 Marbles in a Bag

We have r red marbles, b blue marbles, and g green marbles in the same bag. If we sample marbles with replacement until we get 3 red marbles (not necessarily consecutively), how many blue marbles should we expect to see?