CS 70 Discrete Mathematics and Probability Theory
Spring 2017 Rao

DIS 11a

1 Correlation and Independence

(a) What does it mean for two random variables to be uncorrelated?

(b) What does it mean for two random variables to be independent?

(c) Are all uncorrelated variables independent? Are all independent variables uncorrelated?

2 Covariance

We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $\text{cov}(X_1, X_2)$?

3 LLSE

We have two bags of balls. The fractions of red balls and blue balls in bag A are $2/3$ and $1/3$ respectively. The fractions of red balls and blue balls in bag B are $1/2$ and $1/2$ respectively. Someone gives you one of the bags (unmarked) uniformly at random. Then we draw 6 balls from the same bag with replacement. Let X_i be the indicator random variable that ball i is red. Now, let us define $X = \sum_{1 \leq i \leq 3} X_i$ and $Y = \sum_{4 \leq i \leq 6} X_i$. Find $L(Y \mid X)$. *Hint*: Recall that

$$L(Y \mid X) = \mathbf{E}(Y) + \frac{\text{cov}(X,Y)}{\text{var}(X)}(X - \mathbf{E}(X)).$$