

1 Binomial Variance

Throw n balls into m bins uniformly at random. For a specific ball i , what is the variance of the number of roommates it has (i.e. the number of other balls that it shares its bin with)?

2 Will I Get My Package?

Sneaky delivery guy of some company is out delivering n packages to n customers. Not only does he hand a random package to each customer, he tends to open a package before delivering with probability $1/2$. Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbf{E}(X)$.

(b) Compute the variance $\text{Var}(X)$.

3 Variance

This problem will give you practice using the "standard method" to compute the variance of a sum of random variables that are not pairwise independent (so you cannot use "linearity" of variance).

- (a) A building has n floors numbered $1, 2, \dots, n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each gets off at a uniformly random one of the n floors (independently of everybody else). What is the *variance* of the number of floors the elevator *does not* stop at? (In fact, the variance of the number of floors the elevator *does* stop at must be the same, but the former is a little easier to compute.)

- (b) A group of three friends has n books they would all like to read. Each friend (independently of the other two) picks a random permutation of the books and reads them in that order, one book per week (for n consecutive weeks). Let X be the number of weeks in which all three friends are reading the same book. Compute $\text{Var}(X)$.