CS 70 Discrete Mathematics and Probability Theory
Spring 2017 Rao

DIS 8a

1 Balls and Bins

Throw n balls into n bins.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Give an upper bound on the probability that at least k bins are empty.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?

2 Easter Eggs

You made the trek to Soda for a Spring Break-themed homework party, and every attendee gets to leave with a party favor. You're given a bag with 20 chocolate eggs and 40 (empty) plastic eggs. You pick 5 eggs without replacement.

(a) What is the probability that the first egg you drew was a chocolate egg?

(b) What is the probability that the second egg you drew was a chocolate egg?

(c) Given that the first egg you drew was an empty plastic one, what is the probability that the fifth egg you drew was also an empty plastic egg?

3 Head Count

Consider flipping a fair coin twice.

(a) What is the sample space Ω generated from these flips?

(b) Define a random variable X to be the number of heads. What is the distribution of X?

(c) Define a random variable Y to be 1 if $\omega = (H,T)$ and 0 otherwise. What is the distribution of Y?

(d) Define a third random variable $Z = X + Y$. What is the distribution of Z?

4 Head Count II

Now consider a new coin with $\Pr(\text{Heads}) = 2/5$. We'll flip the coin 20 times.

(a) As before, define X to be the number of heads. What is $\Pr(X = 7)$?

(b) What is $\Pr(X \geq 1)$?

(c) What is $\Pr(12 \leq X \leq 14)$?