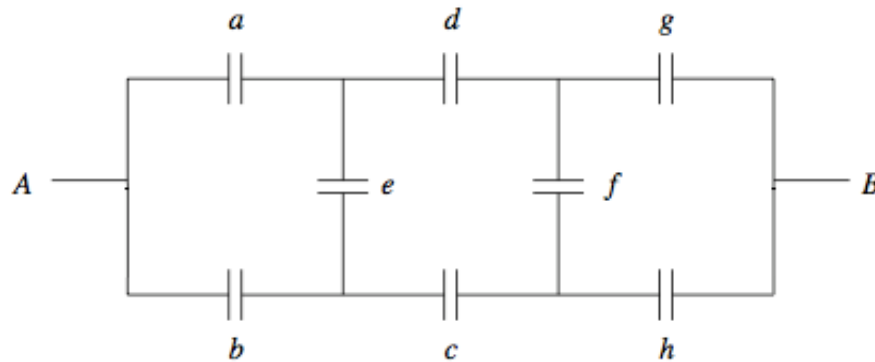


## 1 Let's Talk Probability

- (a) When is  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$  true? What is the general rule that always holds?
- (b) When is  $\Pr(A \cap B) = \Pr(A)\Pr(B)$  true? What is the general rule that always holds?
- (c) If  $A$  and  $B$  are disjoint, are they independent?
- (d) On the space of a fair roll of a six-sided die, find three events, each of which is independent of the intersection of the other two, such that they are not mutually independent.
- (e) If we roll 2 dice, what is the probability that the first roll is a 3? What is the probability that the first roll is a 3 if we know that the sum of the dice is 6?

## 2 Communication Network

In the communication network shown below, link failures are independent, and each link has a probability of failure of  $p$ . Consider the physical situation before you write anything.  $A$  can communicate with  $B$  as long as they are connected by at least one path which contains only in-service links.



- (a) Given that exactly five links have failed, determine the probability that  $A$  can still communicate with  $B$ .
- (b) Given that exactly five links have failed, determine the probability that either  $g$  or  $h$  (*but not both*) is still operating properly.
- (c) Given that  $a$ ,  $d$  and  $h$  have failed (but no information about the information of the other links), determine the probability that  $A$  can communicate with  $B$ .

### 3 Marbles

Box  $A$  contains 1 black and 3 white marbles, and box  $B$  contains 2 black and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

- (a) What is the probability that the marble is black?
- (b) Given that the marble is white, what is the probability that it came from box  $A$ ?

### 4 Lie Detector

A lie detector is known to be  $4/5$  reliable when the person is guilty and  $9/10$  reliable when the person is innocent. If a suspect is chosen from a group of suspects of which only  $1/100$  have ever committed a crime, and the test indicates that the person is guilty, what is the probability that he is innocent?