

1 Bit String

How many bit strings of length 10 contain at least five consecutive 0's?

2 Flippin' Coins

Suppose we have a biased coin, with outcomes H and T , with probability of heads $\Pr[H] = 3/4$ and probability of tails $\Pr[T] = 1/4$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

- (a) What is the *sample space* for our experiment?
- (b) Which of the following are examples of *events*? Select all that apply.
- $\{(H, H, T), (H, H), (T)\}$
 - $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
 - $\{(T, T, T)\}$
 - $\{(T, T, T), (H, H, H)\}$
 - $\{(T, H, T), (H, H, T)\}$
- (c) What is the complement of the event $\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, T, T)\}$?
- (d) Let A be the event that our outcome has 0 heads. Let B be the event that our outcome has exactly 2 heads. What is $A \cup B$?
- (e) What is the probability of the outcome H, H, T ?
- (f) What is the probability of the event that our outcome has exactly two heads?

3 Aces

Consider an ordinary deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

4 Probability Practice

- (a) If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?
- (b) A message source M of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0, 1, 2\}$, and all such words are equally probable. What is the probability that M produces a word that looks like a byte (*i.e.*, no appearance of '2')?
- (c) If five numbers are selected at random from the set $\{1, 2, 3, \dots, 20\}$, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)