CS 70 Discrete Mathematics and Probability Theory
Spring 2017 Rao

DIS 3b

1 Modular Arithmetic

Solve the following equations for x and y modulo the indicated modulus, or show that no solution exists. Show your work.

(a) $9x \equiv 1 \pmod{11}$.

(b) $10x + 23 \equiv 3 \pmod{31}$.

(c) $3x + 15 \equiv 4 \pmod{21}$.

(d) The system of simultaneous equations $3x + 2y \equiv 0 \pmod{7}$ and $2x + y \equiv 4 \pmod{7}$.

2 Baby Fermat

Assume that a does have a multiplicative inverse $\pmod{m}$. Let us prove that its multiplicative inverse can be written as $a^k \pmod{m}$ for some $k \geq 0$.

(a) Consider the sequence $a, a^2, a^3, \ldots \pmod{m}$. Prove that this sequence has repetitions.

(b) Assuming that $a^i \equiv a^j \pmod{m}$, where $i > j$, what can you say about $a^{i-j} \pmod{m}$?

(c) Prove that the multiplicative inverse can be written as $a^k \pmod{m}$. What is k in terms of i and j?

3 Does It Exist?

Can you find a number that is a perfect square and is a multiple of 2 but not a multiple of 4? Either give such a number or prove that no such number exists.

4 Bijections

Let n be an odd number. Let $f(x)$ be a function from $\{0, 1, \ldots, n-1\}$ to $\{0, 1, \ldots, n-1\}$. In each of these cases say whether or not $f(x)$ is necessarily a bijection. Justify your answer (either prove $f(x)$ is a bijection or give a counterexample).

(a) $f(x) = 2x \pmod{n}$.

(b) $f(x) = 5x \pmod{n}$.

(c) n is prime and

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x^{-1} \pmod{n} & \text{if } x \neq 0. \end{cases}$$

(d) n is prime and $f(x) = x^2 \pmod{n}$.