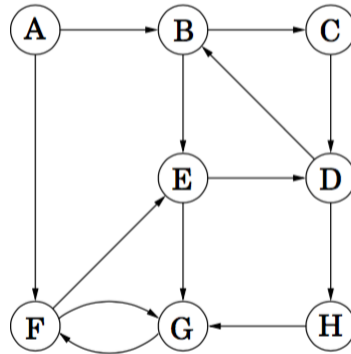


1 Graph Basics

In the first few parts, you will be answering questions on the following graph G .



- (a) What are the vertex and edge sets V and E for graph G ?
- (b) Which vertex has the highest in-degree? Which vertex has the lowest in-degree? Which vertices have the same in-degree and out-degree?
- (c) What are the paths from vertex B to F , assuming no vertex is visited twice? Which one is the shortest path?
- (d) Which of the following are cycles in G ?
- $\{(B,C), (C,D), (D,B)\}$
 - $\{(F,G), (G,F)\}$
 - $\{(A,B), (B,C), (C,D), (D,B)\}$
 - $\{(B,C), (C,D), (D,H), (H,G), (G,F), (F,E), (E,D), (D,B)\}$
- (e) Which of the following are walks in G ?
- $\{(E,G)\}$
 - $\{(E,G), (G,F)\}$
 - $\{(F,G), (G,F)\}$
 - $\{(A,B), (B,C), (C,D)\}$
 - $\{(E,G), (G,F), (F,G), (G,F)\}$
 - $\{(E,D), (D,B), (B,E), (E,D), (D,H), (H,G), (G,F)\}$

(f) Which of the following are tours in G ?

- i. $\{(E, G)\}$
- ii. $\{(E, G), (G, F)\}$
- iii. $\{(F, G), (G, F)\}$
- iv. $\{(A, B), (B, C), (C, D)\}$
- v. $\{(E, G), (G, F), (F, G), (G, F)\}$
- vi. $\{(E, D), (D, B), (B, E), (E, D), (D, H), (H, G), (G, F)\}$

In the following three parts, let's consider a general undirected graph G with n vertices ($n \geq 3$).

- (g) True/False: If each vertex of G has degree at most 1, then G does not have a cycle.
- (h) True/False: If each vertex of G has degree at least 2, then G has a cycle.
- (i) True/False: If each vertex of G has degree at most 2, then G is not connected.

2 Bipartite Graph

Consider an undirected bipartite graph with two disjoint sets L, R . Prove that a graph is bipartite if and only if it no cycles of odd length.

3 Planarity

Consider graphs with the property T : For every three distinct vertices v_1, v_2, v_3 of graph G , there are at least two edges among them. Prove that if G is a graph on ≥ 7 vertices, and G has property T , then G is nonplanar.