1. **Polya suggests: learning is better with friends!**

Solver.

(a) Prepare: comfortable position, pencil, paper, etc.

(b) Read hints, suggestions, discuss with partner.

(c) Read the problem aloud.

(d) Solve on own. You speak, you solve, parter listens.

(e) Speak! No need to choose words.

(f) Go back over problem; "I'm stuck. I better start over." "No that won't work." "Let's see... hmmm."

(g) Try to solve even trivial problems!

Listener.

(a) Listener not a critic. "Please elaborate." "What are you thinking now?" "Can you check that?"

(b) Role: (a) Demand that PS keep talking but don't interrupt. (b) Make sure that PS follows the strategy and doesn't skip any of the steps. (c) Help PS improve his/her accuracy. (d) Help reflect the mental process PS is following. (e) Make sure you understand each step.

(c) Do not turn away from PS and start to work on problem!!!!!

(d) Do not let PS continue if:

   i. You don't understand. "I don't understand." or "I don't follow that."

   ii. When there is a mistake. "Maybe check that." or "Does that sound right?"

(e) No hints! Point out errors, but no correction.

2. **Pigeonhole Principle**

   Prove that if you put $n+1$ apples into n boxes, any way you like, then at least one box must contain at least 2 apples. This is known as the *pigeonhole principle*.

3. **Contraposition**

   Prove that if $a+b < c+d$, then $a < c$ or $b < d$.

4. **Proof by?**

   (a) Prove that if $x, y \in \mathbb{Z}$, if 10 does not divide xy, then 10 does not divide x and 10 does not divide y. In notation: $(\forall x, y \in \mathbb{Z})\ \ 10 \nmid xy \implies (10 \nmid x \ \wedge \ 10 \nmid y)$. What proof technique did you use?

   (b) Prove or disprove the contrapositive.

   (c) Prove or disprove the converse.

5. **Perfect Square**

   A *perfect square* is an integer n of the form $n = m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.

6. **Fermat's Contradiction**

   Prove that $2^{1/n}$ is not rational for any integer $n > 3$. (*Hint*: Use Fermat's Last Theorem.)